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TWO-TIME-SCALE MAGNETIC ATTITUDE CONTROL OF LEO SPACECRAFT

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- A proof of almost global exponential stability is provided, for a proper selection of control gains, in the framework of Singular Perturbation Theory (SPT).
- System robustness is proven in the presence of environmental disturbances, implementation issues, and actuator saturation limits, if the effect of magnetic residual dipoles is mitigated by online estimation.

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where

- $\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{e}}_2$, and $\hat{\boldsymbol{e}}_3$ are principal axes of inertia
- $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^T$ is the absolute angular velocity vector of the spacecraft,
- $\boldsymbol{J} = \mathrm{diag}(J_1, J_2, J_3)$ is the spacecraft inertia matrix,
- $J_2 \neq J_1, J_3$ and $J_1 = J_3$, that is, the spacecraft has axisymmetric inertia properties about $\hat{\boldsymbol{e}}_2$
- $\boldsymbol{M}^{(c)}$, and $\boldsymbol{M}^{(d)}$ are the control and disturbance torques, respectively.

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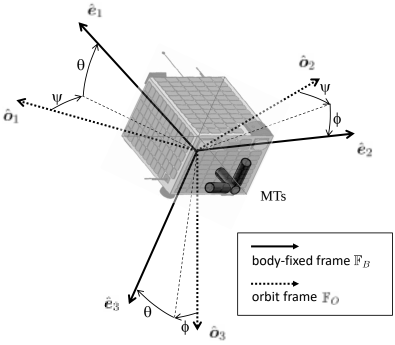

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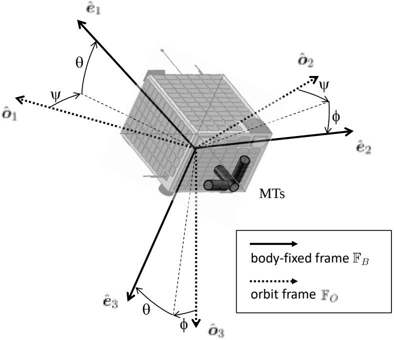

The coordinate transformation matrix between $\mathbb{F}_O$ and $\mathbb{F}_B$, parametrized by a 3-1-2 Euler sequence, is:

$$\boldsymbol{T}_{BO} = \begin{pmatrix} \mathrm{c}\psi\mathrm{c}\theta - \mathrm{s}\phi\mathrm{s}\psi\mathrm{s}\theta & \mathrm{c}\theta\mathrm{s}\psi + \mathrm{c}\psi\mathrm{s}\phi\mathrm{s}\theta & -\mathrm{c}\phi\mathrm{s}\theta \\ -\mathrm{c}\phi\mathrm{s}\psi & \mathrm{c}\phi\mathrm{c}\psi & \mathrm{s}\phi \\ \mathrm{c}\psi\mathrm{s}\theta + \mathrm{c}\theta\mathrm{s}\phi\mathrm{s}\psi & \mathrm{s}\psi\mathrm{s}\theta - \mathrm{c}\psi\mathrm{c}\theta\mathrm{s}\phi & \mathrm{c}\phi\mathrm{c}\theta \end{pmatrix} \tag{4}$$

Euler angles evolve as a function of the angular speed of the spacecraft relative to $\mathbb{F}_O$, given by $\boldsymbol{\omega}^r = \boldsymbol{\omega} - \boldsymbol{T}_{BO}\,\boldsymbol{\omega}_O^{orb}$, where $\boldsymbol{\omega}_O^{orb} = (0, n, 0)^T$.
The kinematics of yaw, roll, and pitch angles is thus written as:

$$\dot{\psi} = (-\omega_1 \sin\theta + \omega_3 \cos\theta + n \sin\phi\cos\psi) \,/\, \cos\phi \tag{5}$$

$$\dot{\phi} = \omega_1 \cos\theta + \omega_3 \sin\theta - n \sin\psi \tag{6}$$

$$\dot{\theta} = \omega_2 + (\omega_1 \sin\phi\sin\theta - \omega_3 \sin\phi\cos\theta - n\cos\psi) \,/\, \cos\phi \tag{7}$$

Let $\hat{\boldsymbol{\sigma}} = \mathbb{T}_{BO}\,(0,1,0)^T$ be the unit vector parallel to the direction of $\hat{\boldsymbol{o}}_2$. Two desired angular momentum vectors are defined:

- $\boldsymbol{h}_d = (0,\eta,0)^T$ (the angular momentum vector becomes parallel to $\hat{\boldsymbol{e}}_2$);
- $\boldsymbol{H}_d = \eta\,\hat{\boldsymbol{\sigma}}$ (the angular momentum becomes parallel to $\hat{\boldsymbol{o}}_2$).

Provided $\lambda > 0$, $\eta : \mathbb{R} \to \mathbb{R}$ is a linear function of θ:

$$\eta(\theta) = J_2\, n\,(1 - \lambda\,\theta) \tag{8}$$

Two different angular momentum error variables are introduced:

$$\boldsymbol{\zeta} = \boldsymbol{H}_d(\theta) - \boldsymbol{J}\,\boldsymbol{\omega} \tag{9}$$

$$\boldsymbol{\varepsilon} = \boldsymbol{h}_d(\theta) - \boldsymbol{J}\,\boldsymbol{\omega} \tag{10}$$

The magnetic control law is:

$$\boldsymbol{M}^{(c)} = \left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^T\right)\,(k_\zeta\,\boldsymbol{\zeta} + k_\varepsilon\,\boldsymbol{\varepsilon}) \tag{11}$$

where k_ζ and k_ε are positive gains and $\hat{\boldsymbol{b}} = \boldsymbol{b}/||\boldsymbol{b}||$.

Let $\boldsymbol{Z} = \mathbb{T}_{BI}^{T}\,\boldsymbol{\zeta}$ and $\boldsymbol{E} = \mathbb{T}_{BI}^{T}\,\boldsymbol{\varepsilon}$:

$$\dot{\boldsymbol{Z}} = -\left[\boldsymbol{T}_{BI}^{T}\left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^{T}\right)\boldsymbol{T}_{BI}\right]\left(k_{\zeta}\,\boldsymbol{Z} + k_{\varepsilon}\,\boldsymbol{E}\right) + \boldsymbol{T}_{BI}^{T}\,\dot{\boldsymbol{H}}_d \tag{12}$$

$$\begin{aligned}\dot{\boldsymbol{E}} = &-\left[\boldsymbol{T}_{BI}^{T}\left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^{T}\right)\boldsymbol{T}_{BI}\right]\left(k_{\zeta}\,\boldsymbol{Z} + k_{\varepsilon}\,\boldsymbol{E}\right)\\ &-\boldsymbol{T}_{BI}^{T}\left[\left(\boldsymbol{J}^{-1}\,\boldsymbol{T}_{BI}\boldsymbol{E}\right)\times\boldsymbol{h}_d\right] + \boldsymbol{T}_{BI}^{T}\,\dot{\boldsymbol{h}}_d\end{aligned} \tag{13}$$

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Given $\boldsymbol{Y} = \left(\boldsymbol{Z}^{T}, \boldsymbol{E}^{T}\right)^{T}$, $\boldsymbol{Y} \in \mathbb{R}^6$, the system in Eqs. (12) and (13) achieves the form

$$\dot{\boldsymbol{Y}} = -\boldsymbol{A}(t)\boldsymbol{K}\,\boldsymbol{Y} - \boldsymbol{B}\left(t, \theta, \boldsymbol{Y}\right) - \boldsymbol{D}\left(t, \theta, \boldsymbol{Y}\right) \tag{14}$$

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where

$$\boldsymbol{A}(t) = \begin{pmatrix} \boldsymbol{T}_{BI}^T \left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^T\right) \boldsymbol{T}_{BI} & \boldsymbol{T}_{BI}^T \left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^T\right) \boldsymbol{T}_{BI} \\ \boldsymbol{T}_{BI}^T \left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^T\right) \boldsymbol{T}_{BI} & \boldsymbol{T}_{BI}^T \left(\boldsymbol{I}_3 - \hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^T\right) \boldsymbol{T}_{BI} \end{pmatrix} \in \mathbb{R}^{6\times 6} \tag{15}$$

is a time-dependent matrix.

$$\boldsymbol{K} = \begin{pmatrix} k_\zeta \, \boldsymbol{I}_3 & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & k_\varepsilon \, \boldsymbol{I}_3 \end{pmatrix} \in \mathbb{R}^{6\times 6} \tag{16}$$

is a gain matrix.

$$\boldsymbol{B}\left(t, \theta, \boldsymbol{Y}\right) = \begin{pmatrix} \boldsymbol{0}_{3\times 1} \\ \boldsymbol{T}_{BI}^T \left[\left(\boldsymbol{J}^{-1} \boldsymbol{T}_{BI} \boldsymbol{E} \right) \times \boldsymbol{h}_d(\theta) \right] \end{pmatrix}, \tag{17}$$

is the gyroscopic coupling term, and

$$\boldsymbol{D}\left(t, \theta, \boldsymbol{Y}\right) = \begin{pmatrix} \boldsymbol{T}_{BI}^T \, \dot{\boldsymbol{H}}_d \\ \boldsymbol{T}_{BI}^T \, \dot{\boldsymbol{h}}_d \end{pmatrix} = \begin{pmatrix} \boldsymbol{I}_3 \\ \boldsymbol{T}_{BI}^T \end{pmatrix} \dot{\boldsymbol{h}}_d \tag{18}$$

is the term related to the time derivative $\dot{\boldsymbol{h}}_d = (0, -\lambda \, J_2 \, n \, \dot{\theta}, 0)^T$.

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is the term related to the time derivative $\dot{\boldsymbol{h}}_d = (0, -\lambda\, J_2\, n\,\dot{\theta}, 0)^T$. Given the definitions of $\boldsymbol{E}$, $\boldsymbol{Z}$, and $\boldsymbol{Y}$, it is:

$$\dot{\theta} = \boldsymbol{Q}\,\left(\boldsymbol{h}_d(\theta) - \boldsymbol{T}_{BI}\,\boldsymbol{S}\,\boldsymbol{Y}\right) - n\,\frac{\cos\psi}{\cos\phi} \tag{19}$$

where

$$\boldsymbol{Q} = (\tan\phi\,\sin\theta/J_1,\; 1/J_2,\; -\cos\theta\,\tan\phi/J_3) \in \mathbb{R}^{1\times3}$$

and $\boldsymbol{S} = (\boldsymbol{0}_{3\times3}\;\, \boldsymbol{I}_3) \in \mathbb{R}^{3\times6}$.

Lemma 1. *Consider the nonlinear time-varying system defined by Eqs.* (14) *and* (19)*. There exist* λ*,* k_ζ*, and* k_ε *such that the origin* $(\boldsymbol{Y}^T, \theta)^T = \mathbf{0}_{7\times 1}$ *is almost-globally exponentially stable.*

Proof: Let $x = \theta$ and $\boldsymbol{z} = \boldsymbol{Y}$ be the vectors containing the slow and the fast variables, respectively. In the standard form:

$$\dot{x} = f(t, x, \boldsymbol{z}, \epsilon) \tag{20}$$

$$\epsilon \, \dot{\boldsymbol{z}} = \boldsymbol{g}(t, x, \boldsymbol{z}, \epsilon) \tag{21}$$

where

$$f(t, x, \boldsymbol{z}, \epsilon) = \boldsymbol{Q} \, (\boldsymbol{h}_d(\theta) - \boldsymbol{T}_{BI} \, \boldsymbol{S} \, \boldsymbol{Y}) - n \, \frac{\cos\psi}{\cos\phi} \tag{22}$$

and

$$\boldsymbol{g}(t, x, \boldsymbol{z}, \epsilon) = -\boldsymbol{A}(t)\boldsymbol{K} \, \boldsymbol{Y} - \boldsymbol{B} \, (t, \theta, \boldsymbol{Y}) - \epsilon \boldsymbol{D} \, (t, \theta, \boldsymbol{Y}) \tag{23}$$

See Theorem 11.4 in 'H.K. Khalil, Nonlinear Systems, Third Edition, Prentice Hall, Upper Saddle River, NJ (2002) Ch. 11'.

Theorem 11.4 *Consider the singularly perturbed system*

$$\dot{x} = f(t, x, z, \varepsilon) \quad (11.47)$$
$$\varepsilon \dot{z} = g(t, x, z, \varepsilon) \quad (11.48)$$

Assume that the following assumptions are satisfied for all

$$(t, x, \varepsilon) \in [0, \infty) \times B_r \times [0, \varepsilon_0]$$

- $f(t, 0, 0, \varepsilon) = 0$ *and* $g(t, 0, 0, \varepsilon) = 0$.
- *The equation*
$$0 = g(t, x, z, 0)$$
has an isolated root $z = h(t, x)$ *such that* $h(t, 0) = 0$.
- *The functions* f, g, h, *and their partial derivatives up to the second order are bounded for* $z - h(t, x) \in B_\rho$.
- *The origin of the reduced system*
$$\dot{x} = f(t, x, h(t, x), 0)$$
is exponentially stable.
- *The origin of the boundary-layer system*
$$\frac{dy}{d\tau} = g(t, x, y + h(t, x), 0)$$
is exponentially stable, uniformly in (t, x).

Then, there exists $\varepsilon^* > 0$ *such that for all* $\varepsilon < \varepsilon^*$, *the origin of* (11.47)–(11.48) *is exponentially stable.* ◇

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Remark 3. The presence of the attitude matrix only affects the evolution in time of the terms $\boldsymbol{B}(t, \theta, \boldsymbol{Y})$ and $\boldsymbol{D}(t, \theta, \boldsymbol{Y})$, influencing the rate of convergence toward the equilibrium, without any consequence on the asymptotic behavior of the closed-loop system.

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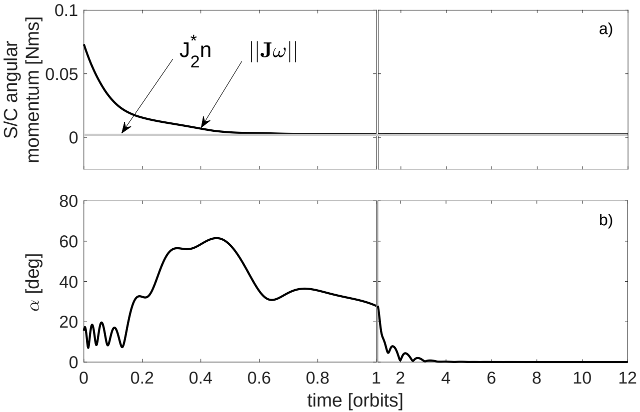
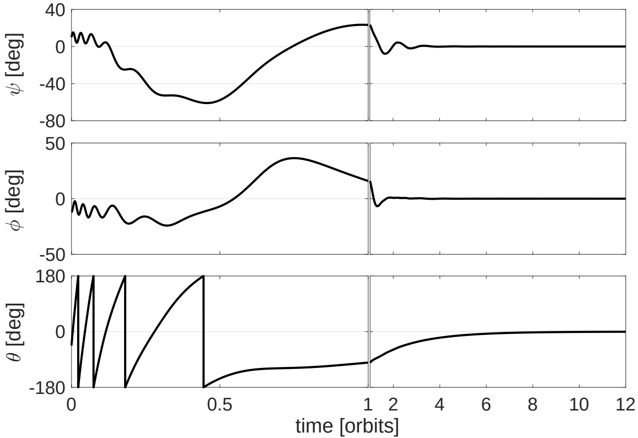
Remark 4. A singularity occurs at $\phi = \pm 90$ deg. From a mathematical standpoint, this implies that the proposed stabilization proof holds almost globally.

Parameter	Symbol	Value	Units
Spacecraft data			
Nominal moments of inertia	$J_1^\star = J_3^\star$	1.416	kg m^2
	$J_2^\star$	2.0861	kg m^2
Maximum control magnetic dipole	$m_{\max}$	3.5	A m^2
Orbit data			
Radius (circular orbit)	r_c	7 021	km
Period	T	5710	s
Inclination	i	98	deg
Right ascension of the ascending node	$RAAN$	137	deg
Sample maneuver			
Initial Conditions	$\boldsymbol{\omega}_0$	$(0.2,\ 2,\ 0.2)^T$	deg/s
	$\psi_0,\ \phi_0,\ \theta_0$	$10, 12, -45$	deg

- $k_\zeta = k_\varepsilon = 0.0009$ s^{-1}, $\lambda = 0.07$ rad^{-1},
- the control dipole is generated as $\boldsymbol{m} = \boldsymbol{m}_c = \left(\hat{\boldsymbol{b}} \times \boldsymbol{M}^{(c)}\right) / \|\boldsymbol{b}\|$,
- Euler angles are bounded as in $-\pi < \psi, \theta \leq +\pi$ and $-\pi/2 < \phi \leq +\pi/2$,
- no disturbance torques, no uncertainties
- ideal measurements, ideal actuation

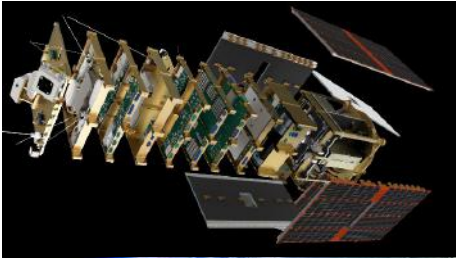


$\alpha = \cos^{-1}(\hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{e}}_2)$ is the angular distance between the desired spin axis $\hat{\boldsymbol{e}}_2$ and the target direction $\hat{\boldsymbol{\sigma}}$



Stabilization of θ: nominal time constant $\tau = 1/(2\pi\lambda) \approx 2$ orbits (theory), effective time constant $\tau \approx 1.9$ orbits (simulation)

Reference spacecraft: ESEO (European Student Earth Orbiter)



Mass distribution uncertainties

- Estimated inertia matrix: $\boldsymbol{J}^{\star} = \mathrm{diag}(1.938, 2.086, 0.894)$ kg m^2
- True spacecraft inertia matrix:

$$\boldsymbol{J} = \begin{pmatrix} 2.0282 & 0.0127 & -0.0016 \\ 0.0127 & 2.0539 & -0.0302 \\ -0.0016 & -0.0302 & 0.8658 \end{pmatrix}$$

Disturbance torques

- Gravity gradient
- Aerodynamic: $\rho = 6.39 \cdot 10^{-13}$ kg/m^3, C_D =2.2, dimensions $L_1 = L_2 = 0.33$ m and $L_3 = 0.66$ m, moment arm $\boldsymbol{r}_{cp} = (0.0082, 0.0030, 0.0492)^T$ m
- Solar radiation pressure: reflectance factor $q_s = 0.8$, $\boldsymbol{r}_{srp} = \boldsymbol{r}_{cp}$, direction of the Sun $\hat{\boldsymbol{s}} = \mathbb{T}_{BI}\,(0.578, 0.578, 0.578)^T$, sunlit area $A_s = \sqrt{A_1^2 + A_2^2 + A_3^2} = 0.33$ m^2
- Residual magnetic dipole: $\boldsymbol{m}_{rm} = (0.15, -0.12, -0.10)^T$ A m^2

Non-ideal sensors modeling

- Angular rate components: standard deviation equal to 0.01 deg/s for the sampled additive white noise signals
- Euler angles: standard deviation equal to 1.07 deg
- Magnetic field components: standard deviation equal to 3 nT, plus a residual bias $(42, -12, -20)^T$ nT

Non-ideal actuation modeling

- Control signals are sampled at a frequency of 1 Hz
- A first-order dynamics with a time constant $\tau_m = 20$ ms is considered (the MTs rise/fall time, calculated as $5\,\tau_m$, is 100 ms)
- A duty-cycle of 800 ms is considered

Control gains

- Closed-loop 'fast' dynamics: $\boldsymbol{k}_\zeta = \boldsymbol{k}_\varepsilon = \mathrm{diag}(0.0069, 0.0138, 0.0230)$ s^{-1}
- Closed-loop 'slow' dynamics: $\lambda = 0.15$ rad^{-1}

Residual dipole estimation

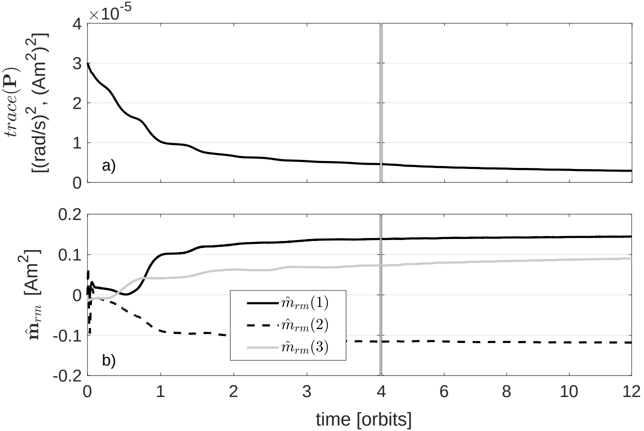
An Extended Kalman Filter, based on the work by Inamori et al.[1] estimates the residual dipole, $\hat{\boldsymbol{m}}_{rm}$.

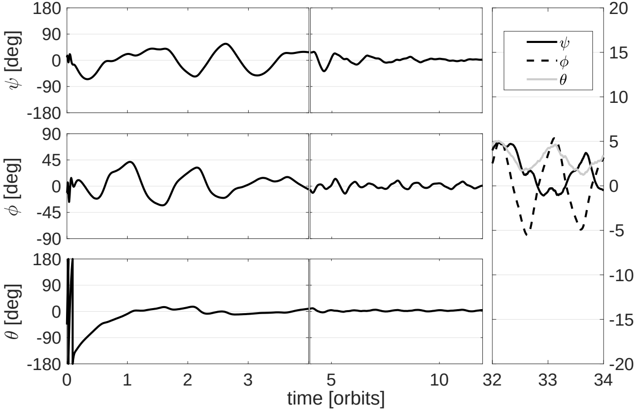
- Estimated state vector at time k: $\hat{\boldsymbol{x}}_k = (\hat{\boldsymbol{\omega}}^T, \hat{\boldsymbol{m}}_{rm}^T)^T\Big|_k \in \mathbb{R}^6$
- Observation vector at time k: $\boldsymbol{z}_k = \boldsymbol{b}_k \in \mathbb{R}^3$
- The prediction phase of the filter is influenced by the input $\boldsymbol{u}_k = \boldsymbol{m}_k \in \mathbb{R}^3$

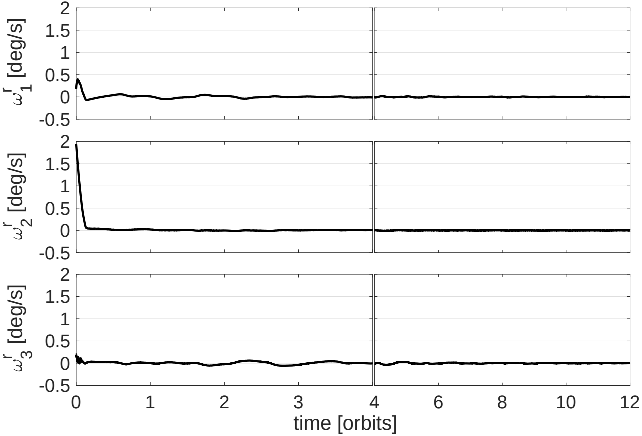
EKF parameters

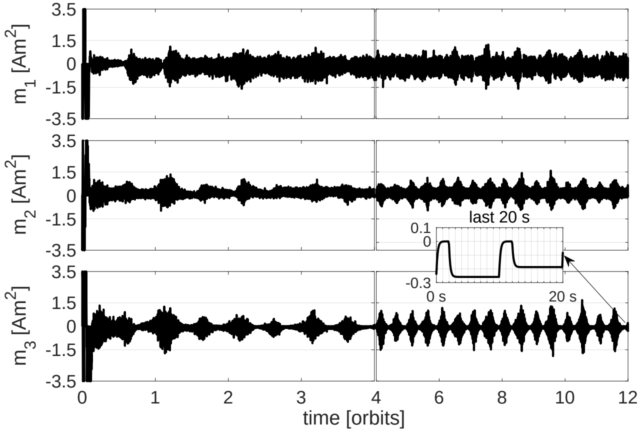
- Update time interval: $\Delta t = t_k - t_{k-1} = 0.1$ s
- EKF initialization: $\hat{\boldsymbol{x}}_0^- = \boldsymbol{0}_{6\times 1}$, $\boldsymbol{P}_0^- = \mathrm{diag}(10^{-9}, 10^{-9}, 10^{-9}, 10^{-5}, 10^{-5}, 10^{-5})$
- Assigned observation noise covariance matrix: $\boldsymbol{R}_k = \boldsymbol{R} = 10^{-8} \cdot \boldsymbol{I}_3$ T^2
- Assigned process noise covariance matrix: $\boldsymbol{Q}_k = \boldsymbol{Q} = 10^{-13} \cdot \boldsymbol{I}_6$

[1] T. Inamori, N. Sako, S. Nakasuka, Magnetic dipole moment estimation and compensation for an accurate attitude control in nano-satellite missions, Acta Astronautica, 68 (2011) 2038-2046.


×10^-5
4
3
2
1
0
$trace(\mathbf{P})$
[(rad/s)^2, (Am^2)^2]
a)
0.2
0.1
0
-0.1
-0.2
$\hat{\mathbf{m}}_{rm}$ [Am^2]
b)
$\hat{m}_{rm}(1)$
$\hat{m}_{rm}(2)$
$\hat{m}_{rm}(3)$
0 1 2 3 4 6 8 10 12
time [orbits]


180
90
0
-90
-180
ψ [deg]
90
45
0
-45
-90
φ [deg]
180
90
0
-90
-180
θ [deg]
0
1
2
3
5
10
32
33
34
time [orbits]
20
15
10
5
0
-5
-10
-15
-20
ψ
φ
θ


2
1.5
1
0.5
0
-0.5
ω_1^r [deg/s]
ω_2^r [deg/s]
ω_3^r [deg/s]
0
1
2
3
4
6
8
10
12
time [orbits]


3.5
1.5
0
-1.5
-3.5
m_1 [Am^2]
m_2 [Am^2]
m_3 [Am^2]
last 20 s
0.1
0
-0.3
0 s
20 s
0
1
2
3
4
6
8
10
12
time [orbits]

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- An extensive simulation campaign can improve the pointing performance by optimal selection of $\boldsymbol{k}_\zeta$, $\boldsymbol{k}_\varepsilon$, and λ.