Attitude Stabilization for Magnetically Actuated Spacecraft using Rotation Matrices

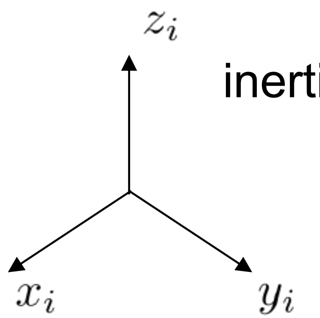
Fabio Celani
School of Aerospace Engineering

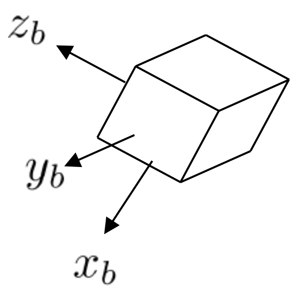
SAPIENZA
UNIVERSITÀ DI ROMA

Outline

- quaternion feedback (Q-F) and the unwinding phenomenon
- rotation matrix feedback (RM-F)
- comparison between Q-F and RM-F for a Cubesat case study

Quaternion and Attitude Rate Feedback





quaternion $(\mathbf{q_v}, q_4)$ $\mathbf{q_v} = [q_1 \;\; q_2 \;\; q_3]^T$ vector part q_4 scalar part

body frame = inertial frame $\Leftrightarrow$ either $(\mathbf{q_v}, q_4) = (\mathbf{0}, 1)$ or $(\mathbf{q_v}, q_4) = (\mathbf{0}, -1)$

for **fully-actuated** spacecraft $\mathbf{T} = -k_p \mathbf{q_v} - k_d \boldsymbol{\omega}$ (**PD-Quat**)

$\Downarrow \; (\mathbf{T}_{disturbance} = \mathbf{0})$

$(\mathbf{q_v}, q_4) = (\mathbf{0}, 1)$ almost globally **asymptotically stable**

$(\mathbf{q_v}, q_4) = (\mathbf{0}, -1)$ **unstable**

Unwinding Phenomenon

$$J_x = J_y = J_z = 1 \text{ kg m}$$

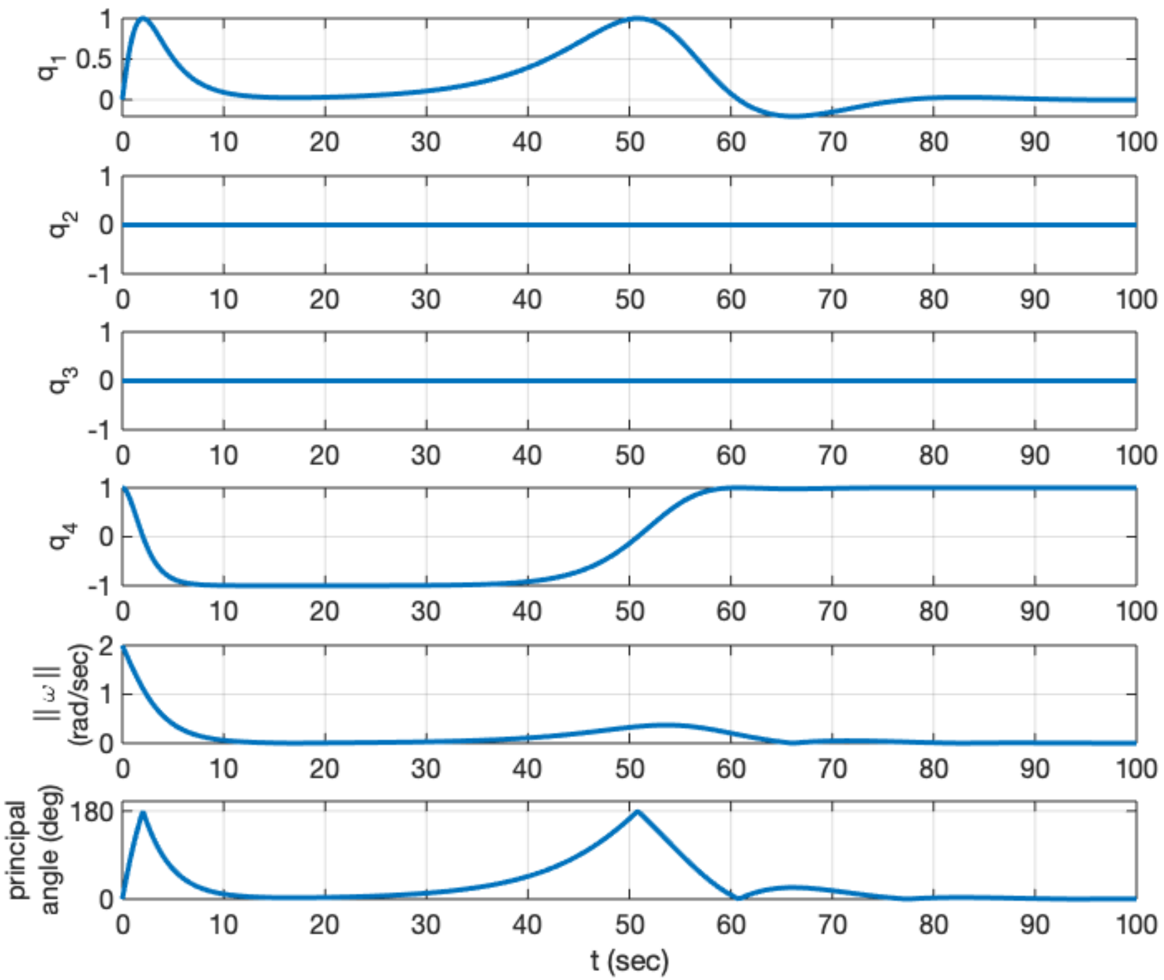
$$k_p = 0.1 \quad k_d = 0.237$$

$$\mathbf{q_v}(0) = \mathbf{0} \quad q_4(0) = 1$$

$$\omega_x(0) = 2 \text{ rad/sec}$$

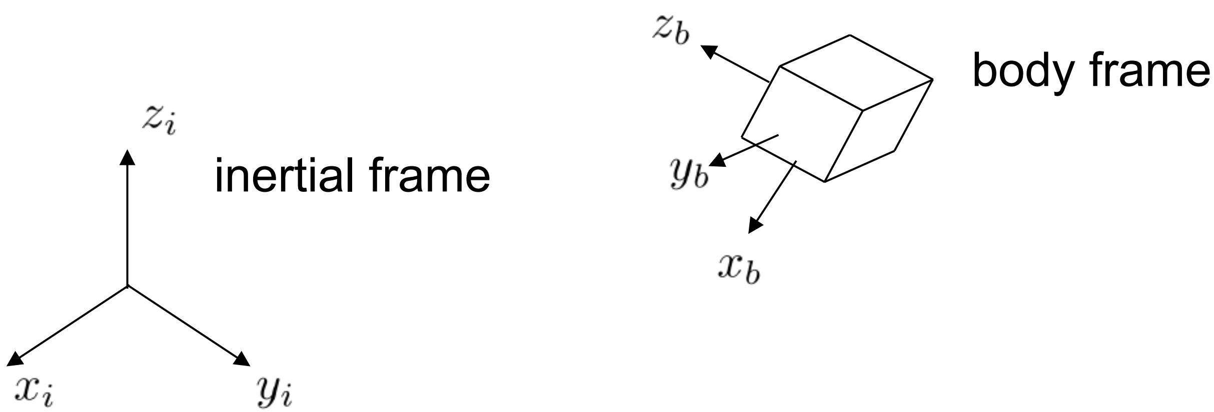
$$\omega_y(0) = \omega_z(0) = 0$$

$$\mathbf{T}_{disturbance} = \mathbf{0}$$



Chaturvedi, Sanyal, McClamroch. Rigid-body attitude control. *Control Systems Magazine. 2011*

Rotation Matrix and Attitude Rate Feedback



rotation matrix $\mathbf{R}$ body frame = inertial frame $\Leftrightarrow \mathbf{R} = \mathbf{I}_{3\times 3}$

for **fully-actuated** spacecraft

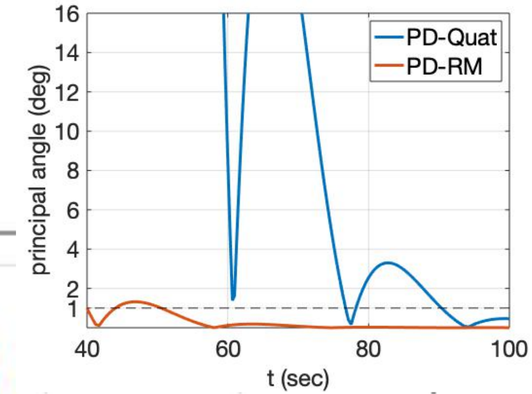
$$\mathbf{T} = -\frac{k_p}{4}\sum_{i=1}^{3}(\mathbf{e_i} \times \mathbf{R}^T \mathbf{e_i}) - k_d \boldsymbol{\omega} \qquad [\mathbf{e_1}\ \mathbf{e_2}\ \mathbf{e_3}] = \mathbf{I}_{3\times 3} \qquad \textbf{(PD-RM)}$$

$$\Downarrow \quad (\mathbf{T}_{disturbance} = \mathbf{0})$$

$\mathbf{R} = \mathbf{I}_{3\times 3}$ almost globally **asymptotically stable**

Chaturvedi, Sanyal, McClamroch. Rigid-body attitude control. *Control Systems Magazine. 2011*

PD-Quat vs PD-RM



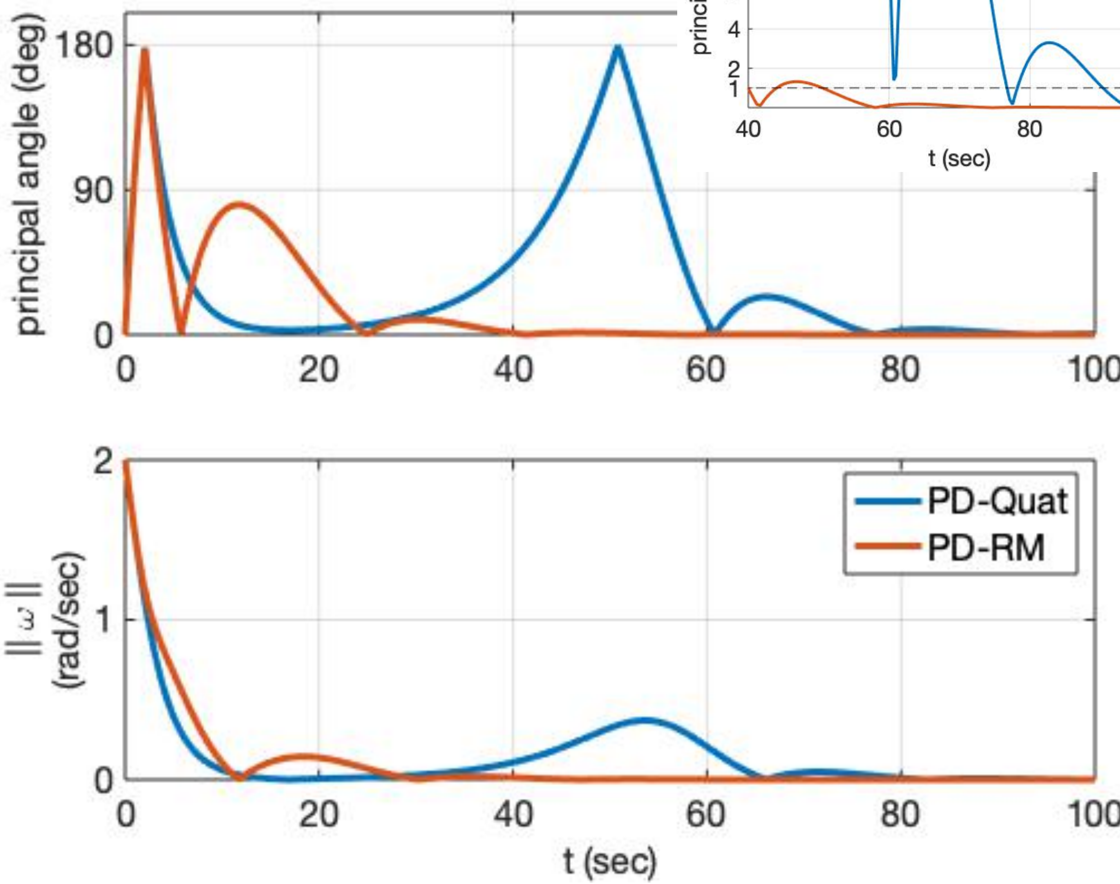
$$J_x = J_y = J_z = 1 \text{ kg m}^2$$

$$k_p = 0.1 \quad k_d = 0.237$$

$$\mathbf{q_v}(0) = \mathbf{0} \quad q_4(0) = 1$$

$$\omega_x(0) = 2 \text{ rad/sec}$$
$$\omega_y(0) = \omega_z(0) = 0$$

$$\mathbf{T}_{disturbance} = \mathbf{0}$$



$$\text{settling time} = \begin{cases} 90.4 \text{ sec} & \mathbf{PD-Quat} \\ 50.8 \text{ sec} & \mathbf{PD-RM} \quad (-44\%) \end{cases}$$

PD-Quat vs PD-RM

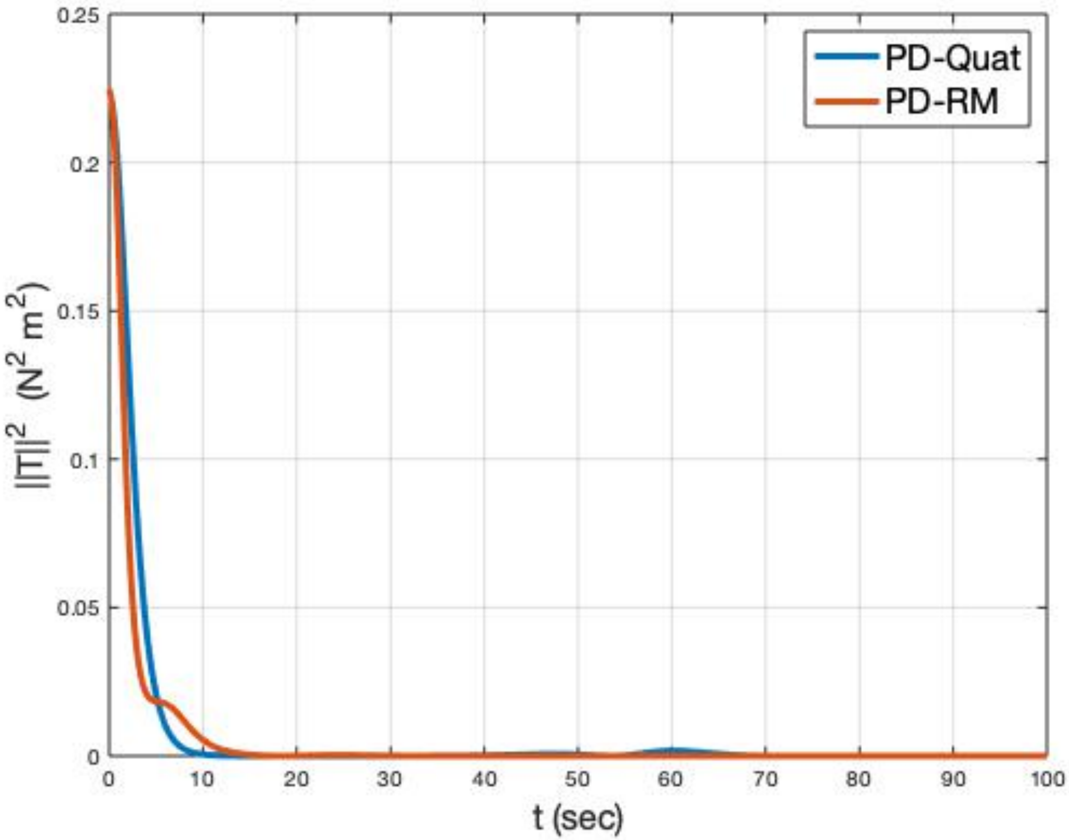
$$J_x = J_y = J_z = 1 \text{ kg m}^2$$

$$k_p = 0.1 \quad k_d = 0.237$$

$$\mathbf{q_v}(0) = \mathbf{0} \quad q_4(0) = 1$$

$$\omega_x(0) = 2 \text{ rad/sec}$$
$$\omega_y(0) = \omega_z(0) = 0$$

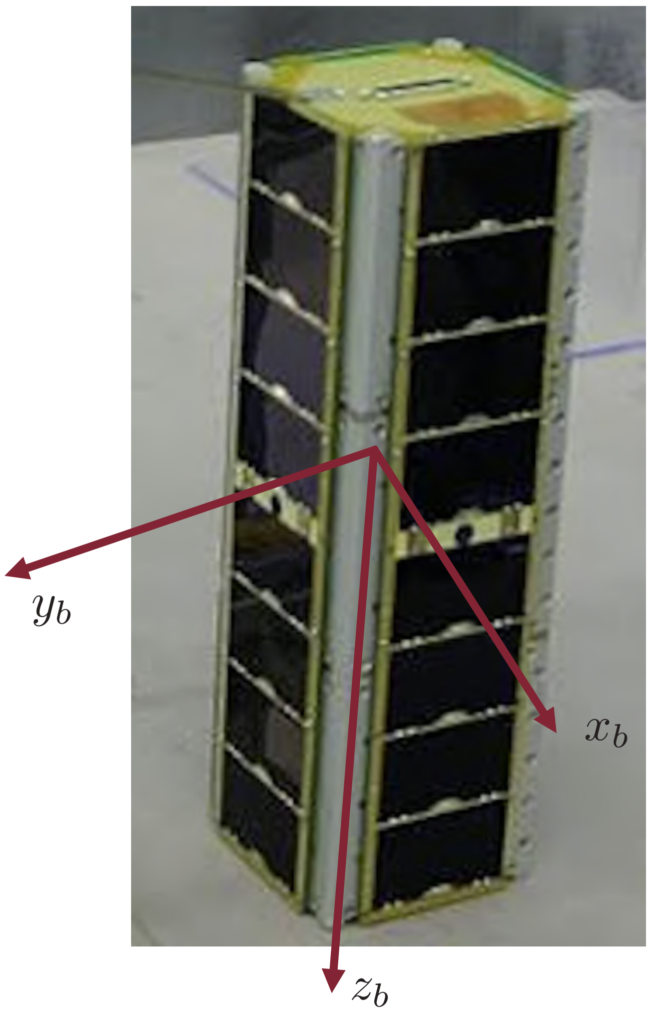
$$\mathbf{T}_{disturbance} = \mathbf{0}$$



energy consumption $\approx \int_0^{t_{fin}} \|\mathbf{T}(t)\|^2 dt = \begin{cases} 0.62 \text{ N}^2 \text{ m}^2 \text{ sec} & \mathbf{PD-Quat} \\ 0.51 \text{ N}^2 \text{ m}^2 \text{ sec} & \mathbf{PD-RM} \quad (-18\%) \end{cases}$

Back to Cubesat World

Tigrisat

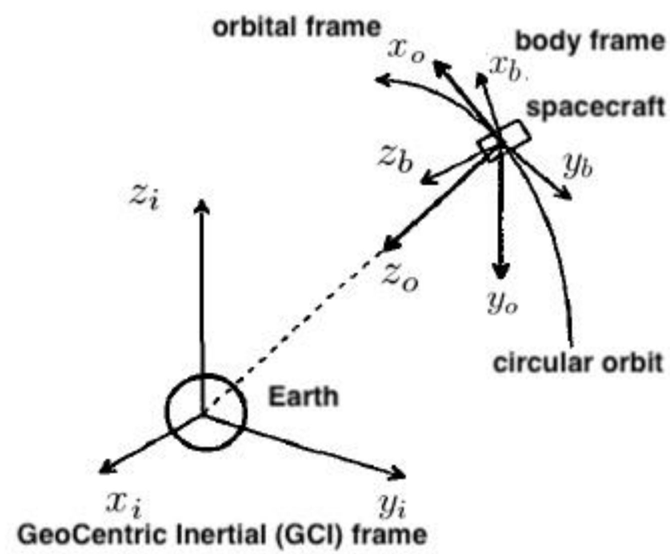


$J_x = J_y = 4.09 \cdot 10^{-2}$ kg m^2 $\quad J_z = 6.5 \cdot 10^{-3}$ kg m^2

circular orbit altitude = 629 km $T_{orbit} = 5832$ sec

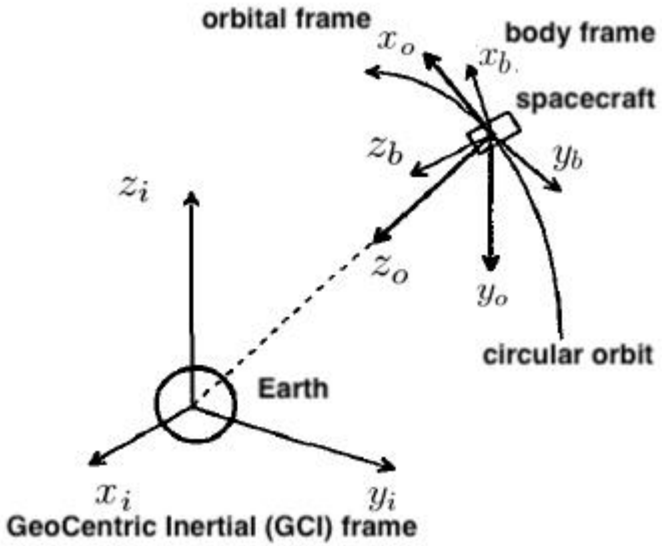
inclination = 97° RAAN = 68.5°

camera pointing along z_b 3 orthogonal magnetorquers



objective: stabilize attitude so that body frame is aligned with orbital frame (camera pointing to Earth)

Quaternion and Attitude Rate Feedback



quaternion $(\mathbf{q_v}, q_4)$

vector part $\mathbf{q_v} = [q_1 \; q_2 \; q_3]^T$ scalar part q_4

body frame = orbital frame

$$\Updownarrow$$

either $(\mathbf{q_v}, q_4) = (\mathbf{0}, 1)$ or $(\mathbf{q_v}, q_4) = (\mathbf{0}, -1)$

$\mathbf{B}$ geomagnetic field at spacecraft

$$\mathbf{m}_{coils} = -\mathbf{B} \times (K_p \mathbf{q_v} + K_d \boldsymbol{\omega}_{bo})$$

(MPD-Quat)

$$K_p = \begin{bmatrix} 293.4863 & 0.5515 & -9.7049 \\ -0.0069 & 299.8118 & -4.1120 \\ 4.8505 & -0.1118 & 299.8613 \end{bmatrix}$$

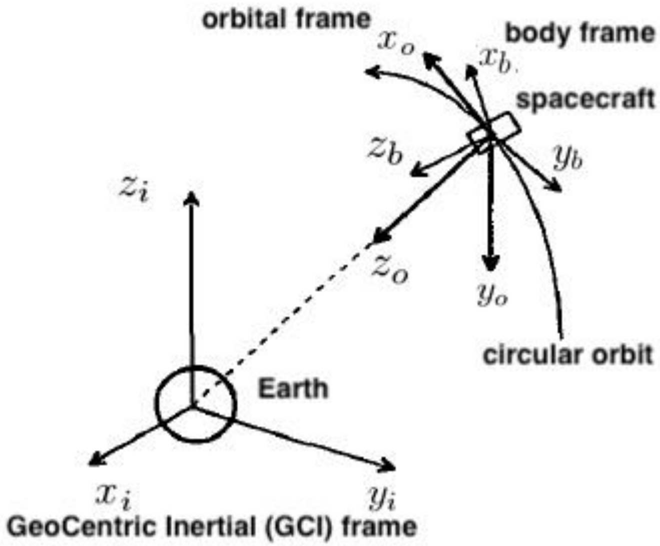
$$K_d = \begin{bmatrix} 1.8 \cdot 10^4 & 0 & 0 \\ 0 & 1.8 \cdot 10^4 & 0 \\ 0 & 0 & 1.8 \cdot 10^4 \end{bmatrix}$$

$$\Downarrow \; (\mathbf{T}_{disturbance} = \mathbf{0})$$

$(\mathbf{q_v}, q_4) = (\mathbf{0}, 1)$ locally **asymptotically stable**

$(\mathbf{q_v}, q_4) = (\mathbf{0}, -1)$ **unstable** $\Rightarrow$ unwinding phenomenon

Rotation Matrix and Attitude Rate Feedback



rotation matrix $\mathbf{R}$

body frame = orbital frame

$$\Updownarrow$$

$$\mathbf{R} = \mathbf{I}_{3\times 3}$$

$\mathbf{B}$ geomagnetic field at spacecraft

$$\mathbf{m}_{coils} = -\mathbf{B} \times \left[\frac{K_p}{4} \sum_{i=1}^{3} (\mathbf{e_i} \times \mathbf{R}^T \mathbf{e_i}) + K_d \boldsymbol{\omega} \right]$$

$$[\mathbf{e}_1 \; \mathbf{e}_2 \; \mathbf{e}_3] = \mathbf{I}_{3\times 3}$$

(MPD-RM)

$$K_p = \begin{bmatrix} 293.4863 & 0.5515 & -9.7049 \\ -0.0069 & 299.8118 & -4.1120 \\ 4.8505 & -0.1118 & 299.8613 \end{bmatrix}$$

$$K_d = \begin{bmatrix} 1.8 \cdot 10^4 & 0 & 0 \\ 0 & 1.8 \cdot 10^4 & 0 \\ 0 & 0 & 1.8 \cdot 10^4 \end{bmatrix}$$

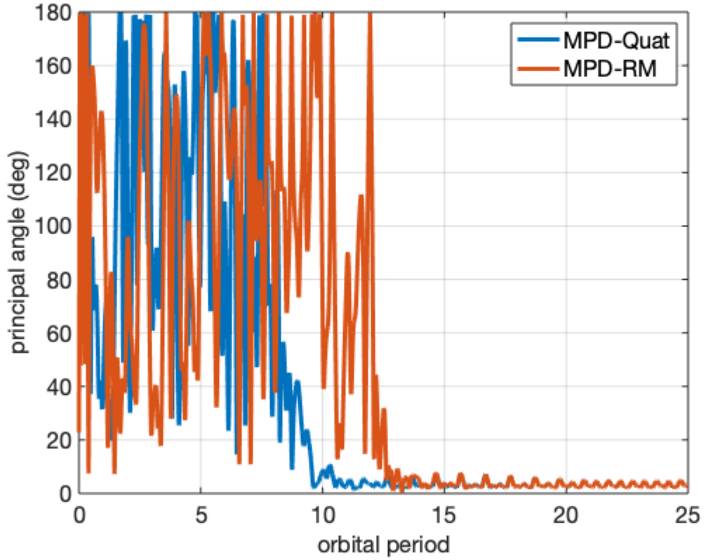
$$\Downarrow \quad (\mathbf{T}_{disturbance} = \mathbf{0})$$

$\mathbf{R} = \mathbf{I}_{3\times 3}$ locally **asymptotically stable** (no unwinding phenomenon)

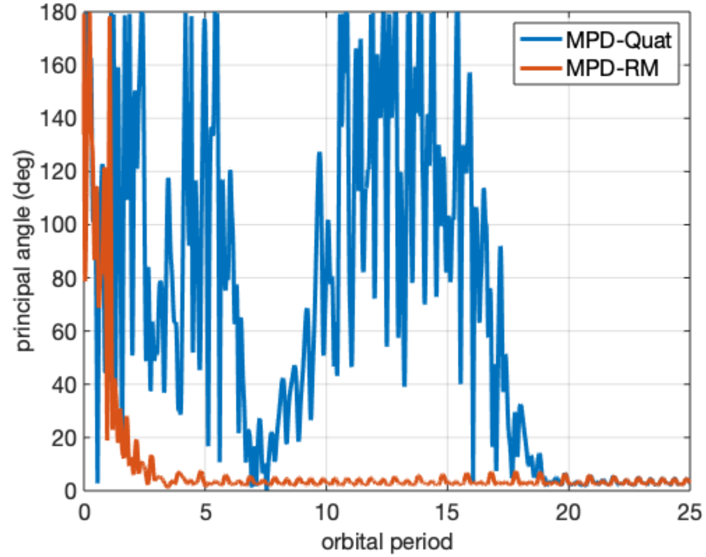
MPD-Quat vs MPD-RM

$$\mathbf{T}_{disturbance} \neq \mathbf{0}$$

initial condition 1



initial condition 2



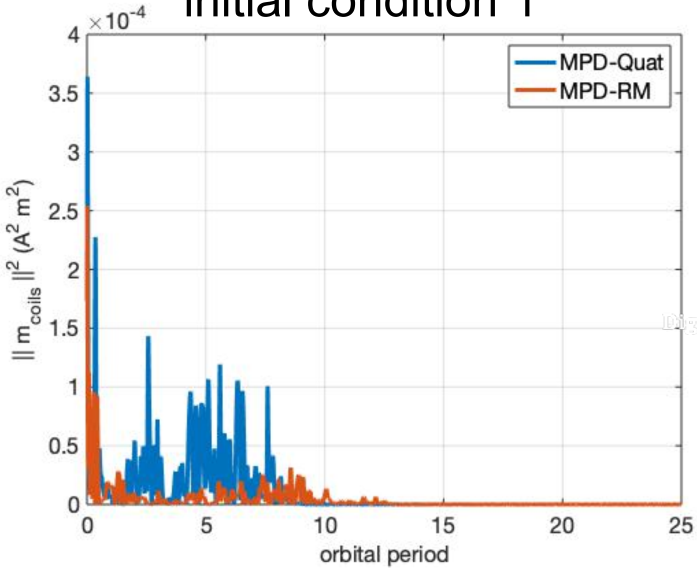
Integral Time Absolute Error (ITAE) $= \int_0^{t_{fin}} t \text{ principal angle}(t) \, dt$

	initial condition 1	**initial condition 2**
MPD-Quat	1.70 10^{11} deg sec^2	5.15 10^{11} deg sec^2
MPD-RM	2.89 10^{11} deg sec^2 (+70%)	0.04 10^{11} deg sec^2 (-99%)

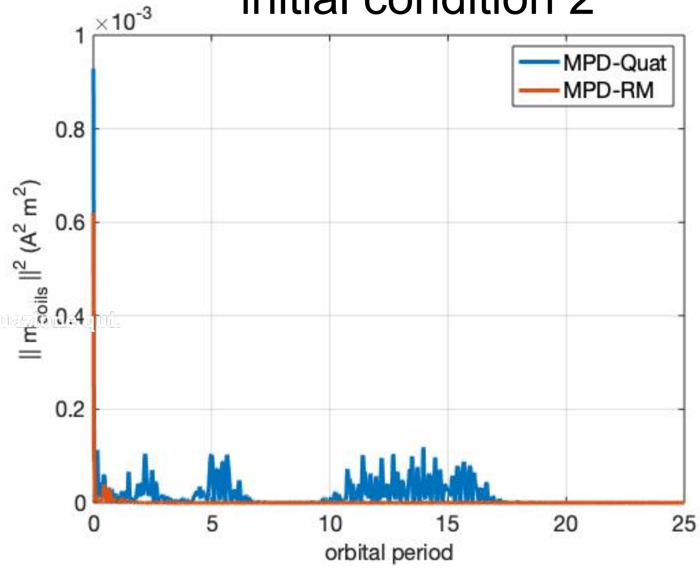
MPD-Quat vs MPD-RM

$$\mathbf{T}_{disturbance} \neq \mathbf{0}$$

initial condition 1



initial condition 2



"energy" consumption $\approx \int_0^{t_{fin}} \|\mathbf{m}_{coils}(t)\|^2 dt$

	initial condition 1	**initial condition 2**
MPD-Quat	1.46 A^2 m^2 sec	2.31 A^2 m^2 sec
MPD-RM	0.54 A^2 m^2 sec (-63%)	0.13 A^2 m^2 sec (-94%)

MPD-Quat vs MPD-RM

Monte Carlo campaign
1000 simulation runs
random initial attitude
random $\boldsymbol{\omega}(0)$ with $\|\boldsymbol{\omega}(0)\| \leq 20 \text{ deg/sec}$

results

mean ITAE MPD-Quat = **2.56 10^{11} deg sec^2**
mean ITAE MPD-RM = **2.34 10^{11} deg sec^2 (-9%)**
number of runs in which (ITAE MPD-RM < ITAE MPD-Quat) = **49%**

mean “energy” consumption MPD-Quat = **9.94 A^2 m^2 sec**
mean “energy” consumption MPD-RM = **8.71 A^2 m^2 sec (-12%)**
number of runs in which (“energy” MPD-RM < “energy” MPD-Quat) = **96%**

Conclusion

- comparison between MPD-Quat and MPD-RM attitude control laws for a CubeSat
- Monte Carlo campaign shows that the two control laws are comparable in terms of speed of convergence
- Monte Carlo campaign shows that MPD-RM leads to lower "energy" consumption

MPD-Quat vs MPD-RM

linearizations

$$\mathbf{m}_{coils} = -\mathbf{B} \times (K_p \mathbf{q_v} + K_d \boldsymbol{\omega}_{bo})$$ **(MPD-Quat)**

$\Downarrow$ linearization about $(\mathbf{q_v}, q_4) = (\mathbf{0}, 1)$

$$\mathbf{m}_{coils} = -\mathbf{B} \times \left[\frac{K_p}{2} \zeta + K_d \boldsymbol{\omega} \right]$$

$\zeta = [\phi \ \theta \ \psi]^T$

3-2-1 Euler angles

$\Uparrow$ linearization about $\mathbf{R} = \mathbf{I}_{3\times3}$

$$\mathbf{m}_{coils} = -\mathbf{B} \times \left[\frac{K_p}{4} \sum_{i=1}^{3} (\mathbf{e_i} \times \mathbf{R}^T \mathbf{e_i}) + K_d \boldsymbol{\omega} \right]$$

(MPD-RM)

initial condition close to desired attitude