Station-keeping about Sun-Mars three-dimensional quasi-periodic Collinear Libration Point Trajectories

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5th IAA Conference on University Satellite Missions and CubeSat Workshop

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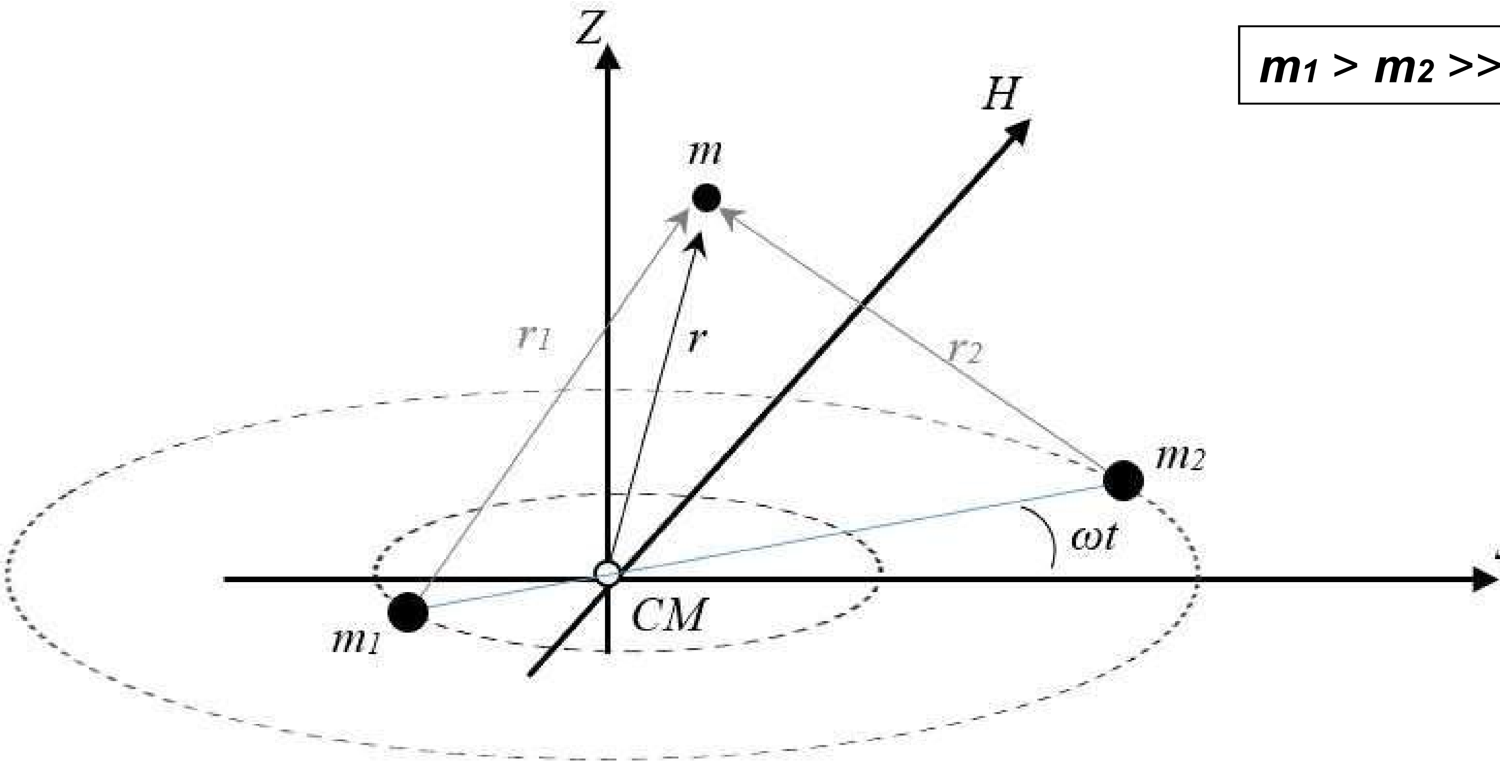
Why collinear libration point orbits

CR3BP: Consider two celestial bodies $m_1 > m_2$ (i.e. Sun & Mars) whose relative motion is described by a **circular orbit** about their center of mass and a **third body** m **of negligible mass** (i.e. a spacecraft) moving under their gravitational influence

$m_1 > m_2 >> m$

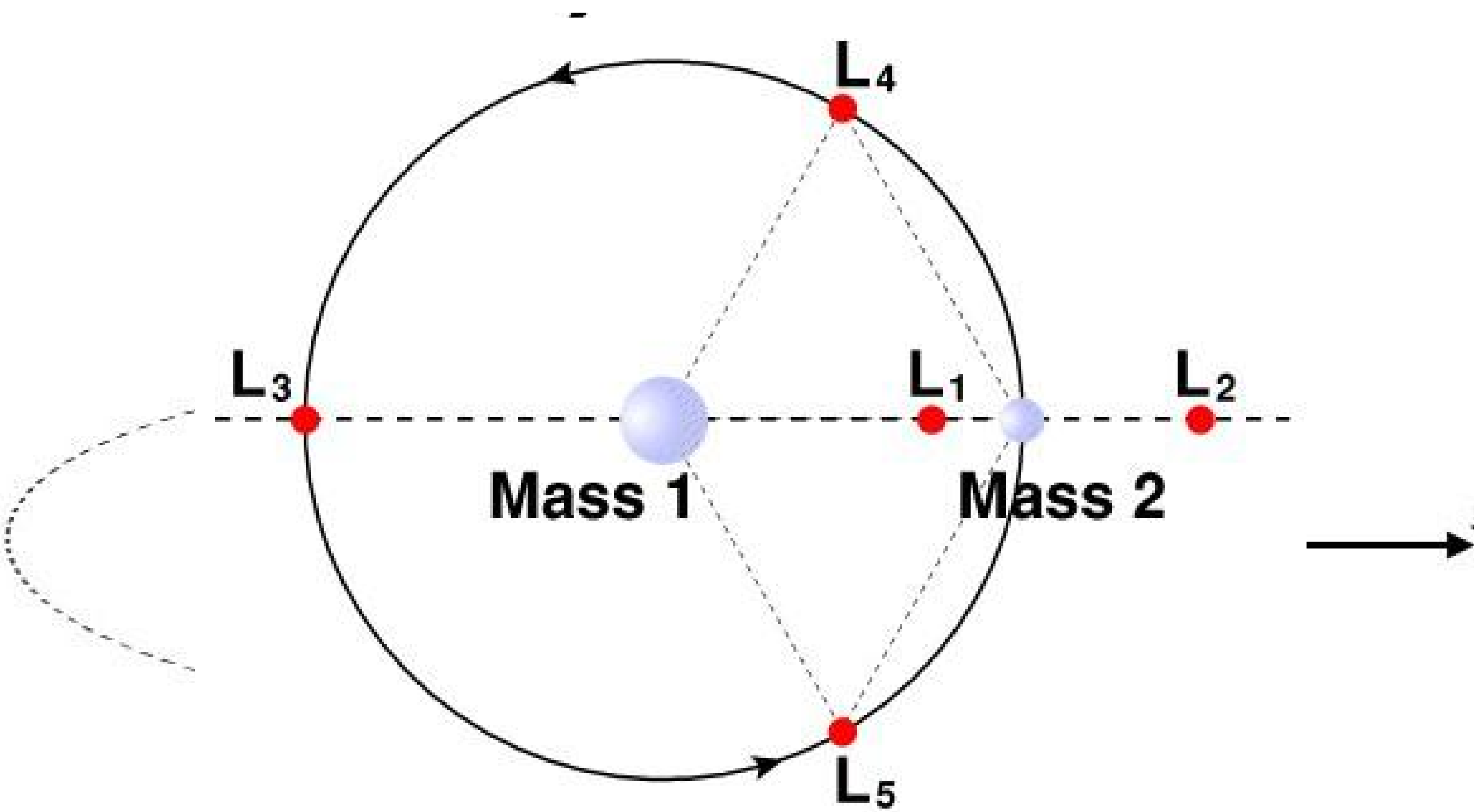


Sketch of the circular restricted 3-body problem geometry in an inertial frame

Why collinear libration point orbits

The dynamics of the CR3BP can be conveniently studied in a frame (synodic) rotating with the celestial bodies m_1 and m_2.

A total of 5 **equilibrium points** exist for the CR3BP. 3 of them (L_1, L_2 and L_3) are located along the straight line connecting m_1 and m_2.

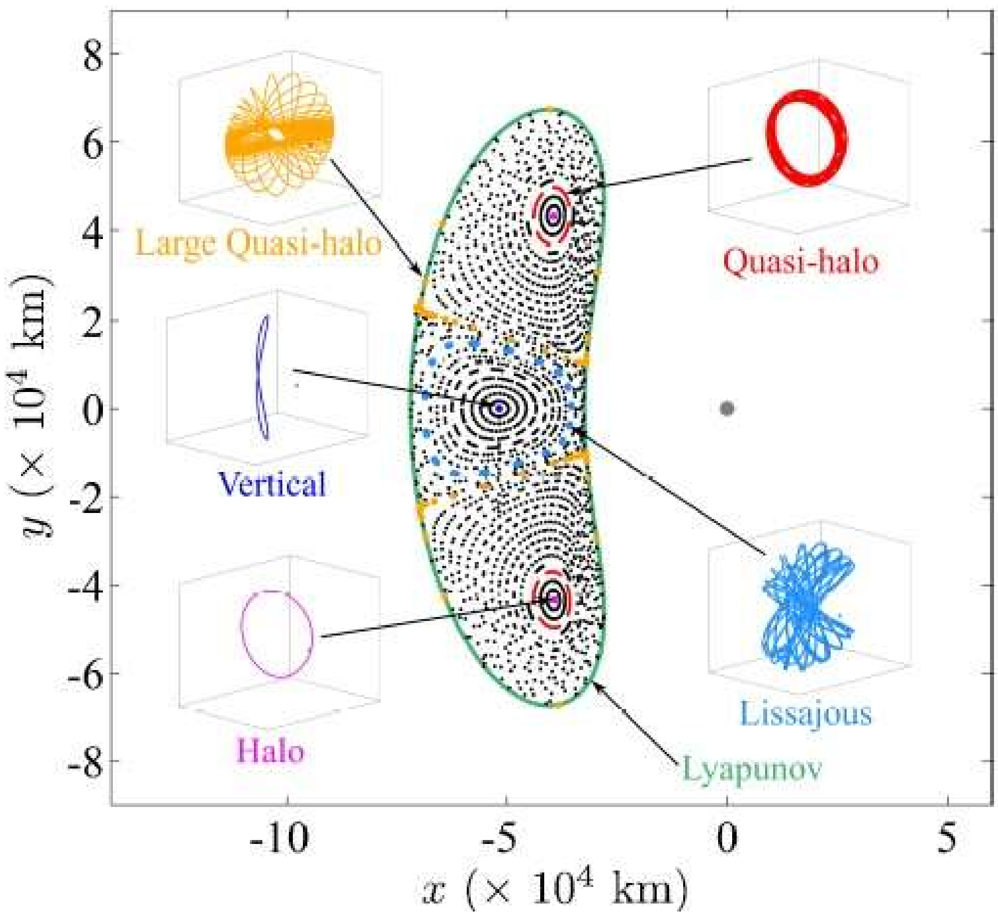


Not to scale.

Sketch of the ci[redacted]*he synodic frame*

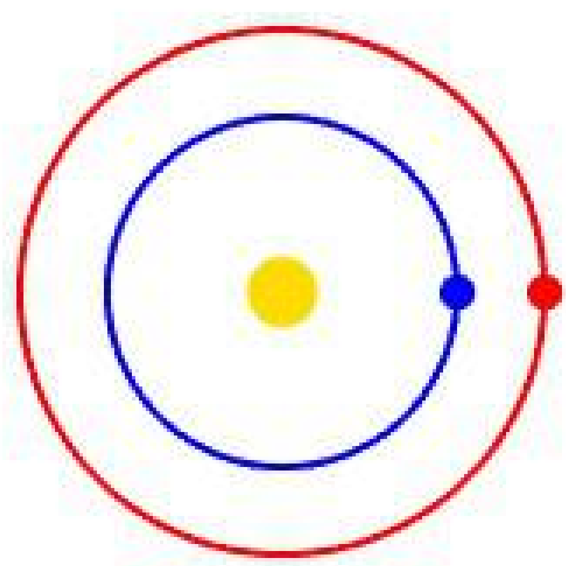
Why collinear libration point orbits

Different types of Quasi-periodic orbits exist in the surrounding of the libration points. In this work, we target low-energy Lissajous orbits.

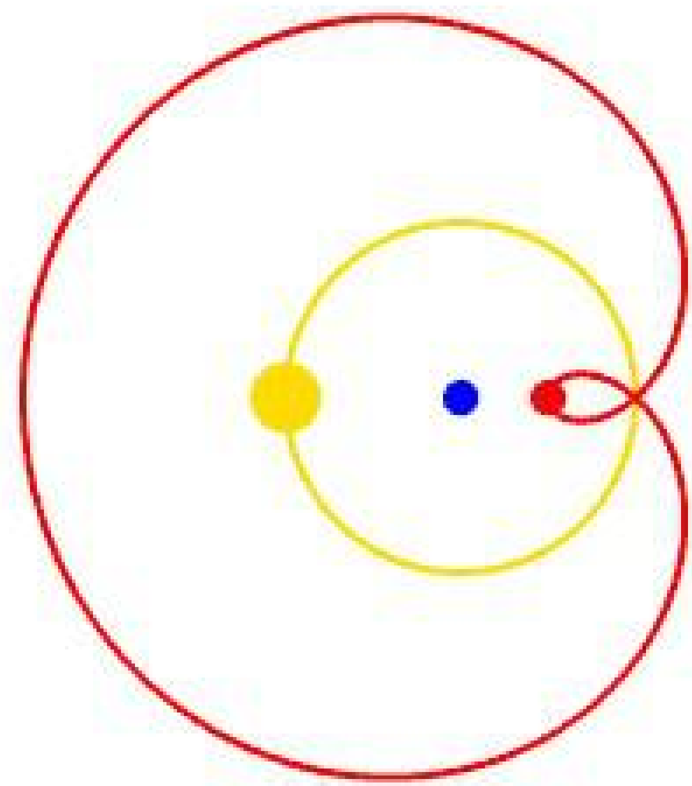


Different families of quasi-periodic collinear libration point orbits

Why collinear libration point orbits



Heliocentric orbits of the Earth and Mars



Apparent Mars retrograde motion (Earth centered)

Why collinear libration point orbits

- **Low environmental disturbances**
- **High observing efficiency**
- **Extended view of the Earth (L_1)**
- **Constant view of the Sun (L_1)**
- **Thermal stability and low temperature (L_2)**
- **Communications relay (L_1, L_2, L_3)**

The Sun-Mars “eccentric” problem

The orbit of Mars around the Sun is quite eccentric

System	Eccentricity
Sun – Mars	0.0934
Earth – Moon	0.0549
Sun – Jupiter	0.0489
Sun – Earth	0.0167
Jupiter – Galilean moons	0.0011 – 0.0094

Elliptic

~~Circular~~ restricted 3-body problem dynamics

$$\begin{cases} \ddot{\Xi} = -G\left\{\frac{m_1[\Xi-\Xi_1(\theta)]}{R_1^3} + \frac{m_2[\Xi-\Xi_2(\theta)]}{R_2^3}\right\} \\ \ddot{H} = -G\left\{\frac{m_1[H-H_1(\theta)]}{R_1^3} + \frac{m_2[H-H_2(\theta)]}{R_2^3}\right\} \\ \ddot{Z} = -G\left\{\frac{m_1[Z-Z_1(\theta)]}{R_1^3} + \frac{m_2[Z-Z_2(\theta)]}{R_2^3}\right\} \end{cases} \rightarrow \begin{cases} x' = v_x \\ y' = v_y \\ z' = v_z \\ v_x' = 2v_y + \tau\left(\frac{\partial u}{\partial x} + x\right) \\ v_y' = -2v_x + \tau\left(\frac{\partial u}{\partial y} + y\right) \\ v_z' = \tau\left(\frac{\partial u}{\partial z} + z\right) - z \end{cases}$$

$$\tau = \frac{1}{1 + e\cos\theta}$$

Hamiltonian formalism and normal forms

The equilibrium points do not exist for the ER3BP. The problem is solved arranging the ER3BP dynamics in a form which is equivalent to that of the CR3BP

Hamiltonian formalism for the ER3BP

$$\begin{cases} q_i' = \dfrac{\partial H}{\partial p_i} \\ p_i' = -\dfrac{\partial H}{\partial q_i} \end{cases} \qquad \begin{matrix} [q_1 \quad q_2 \quad q_3]^T = [x \quad y \quad z] \\ [p_1 \quad p_2 \quad p_3]^T = [v_x - y \quad v_y + x \quad v_z] \end{matrix}$$

Hamiltonian function for the ER3BP

$$H = \frac{1}{2}(q_1^2 + q_2^2 + p_1^2 + p_2^2 + p_3^2 + 2p_1q_2 - 2p_2q_1) + \frac{q_3^2}{2} - \tau\left[\sum_{i=1}^{3}\frac{\mu_i}{r_i} + \frac{1}{2}(q_1^2 + q_2^2 + q_3^2)\right]$$

Hamiltonian formalism and normal forms

The Hamiltonian function is expanded in power series to isolate the term depending on the eccentricity. The resulting function is the sum of the Hamiltonian for the CR3BP and a perturbation term.

Hamiltonian for the ER3BP expanded about *e=0*

$$F(\boldsymbol{q},\boldsymbol{p},e) = \tau\left[\sum_{i=1}^{3}\frac{\mu_i}{r_i} + \frac{1}{2}(q_1^2+q_2^2+q_3^2)\right] = F^* + e\frac{\partial F}{\partial e}\bigg|^* + o(e)$$

$$F^* = \frac{1}{2}(p_1^2+p_2^2+p_3^2) + (p_1q_2 - p_2q_1) - \left(\frac{1-\mu}{r_1}+\frac{\mu}{r_2}\right)$$

Hamiltonian formalism and normal forms

After isolating the perturbation terms, the expanded Hamiltonian (*F*) is linearized about a collinear libration point of the CR3BP (i.e. L_1)

Linear Hamiltonian for the ER3BP expanded about *e=0*

$$\boxed{H_2 = \frac{1}{2}\left(p_1^2 + p_2^2 + p_3^2 + 2p_1q_2 - 2p_2q_1\right)} - \left(C_1(\theta) + C_2(\theta)\right)\left(q_1^2 - \frac{q_2^2}{2} - \frac{q_3^2}{2}\right) +$$

$$-e\cos\theta\left\{\left(C_1(\theta) + C_2(\theta)\right)\left(q_1^2 - \frac{q_2^2}{2} - \frac{q_3^2}{2}\right) + \frac{1}{2}\left(q_1^2 + q_2^2 + q_3^2\right)\right\} +$$

$$-e\cos\theta\left\{-q_1D_1\left(C_2(\theta) - C_1(\theta)\right) + D_2\left(C_3(\theta) + C_4(\theta)\right)\left(q_1^2 - \frac{5q_2^2}{2} - \frac{5q_3^2}{2}\right)\right\}$$

Hamiltonian formalism and normal forms

A canonical transformation is defined through the generating function *S*, absorbing the perturbation terms above the second order

Generating function

$$\begin{cases} p_i = \dfrac{\partial S}{\partial q_i} \\ Q_i = \dfrac{\partial S}{\partial P_i} \end{cases} \qquad \widetilde{H}(Q_i, P_i, \theta_b, \theta) = H(q_i, p_i, \theta_b, \theta) + \frac{\partial S}{\partial(\theta_b - \theta)} \frac{\partial(\theta_b - \theta)}{\partial \theta} + \frac{\partial S}{\partial \theta}$$

Linear Hamiltonian in the new normal form

$$H_2 = \frac{1}{2}(P_1^2 + P_2^2 + P_3^2) + P_1 Q_2 - P_2 Q_1 - \left(\frac{\mu}{|L_x + \mu|^3} + \frac{1-\mu}{|L_x + \mu - 1|^3}\right)\left(Q_1^2 - \frac{1}{2}Q_2^2 - \frac{1}{2}Q_3^2\right) = h$$

H_2 is equivalent to the linear Hamiltonian function for the CR3BP. The **complexity** introduced by the more accurate model (*e*) were **absorbed by the transformation**.

Hamiltonian formalism and normal forms

Applying a final canonical transformation by Siegel and Moser, H_2 can be expressed as the sum of three local integrals of motion.

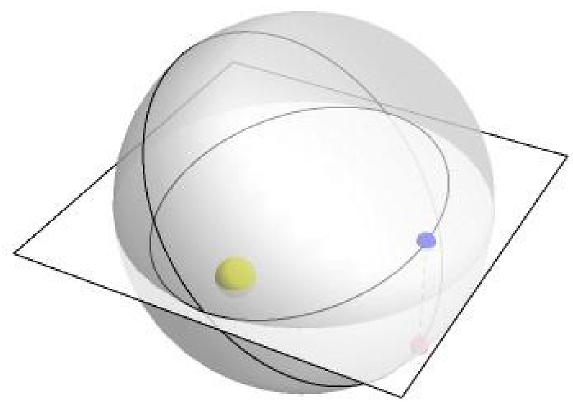
Siegel-Moser transformation

$$\begin{bmatrix} x_1 \\ x_2 \\ x_2 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \boldsymbol{T_N} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

- $\boldsymbol{T_N}$ depends only on ρ $\lambda 1$ and $\lambda 2$ (eigenvalues)
- (x_1, y_1) real and only depend on x, y, u and v
- (x_2, y_2) complex and only depend on x, y, u and v
- (x_3, y_3) complex and only depend on z and w

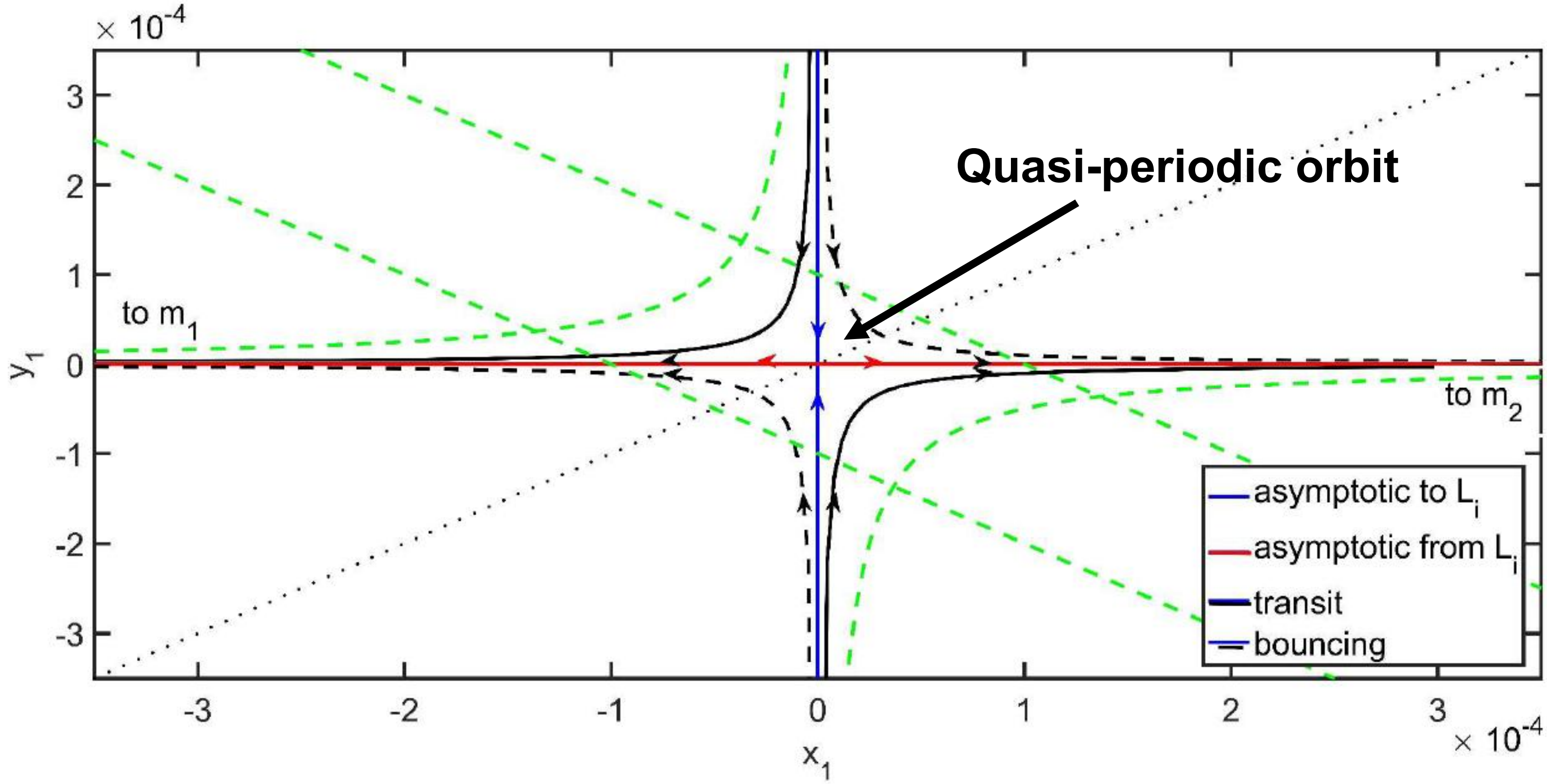
$$H_2 = \rho x_1 y_1 + \frac{\lambda_1}{2}(x_2^2 + y_2^2) + \frac{\lambda_2}{2}(x_3^2 + y_3^2) = h$$

- Unstable equilibrium or drift term (in-plane)
- Center or harmonic oscillator (in-plane)
- Center or harmonic oscillator (out of plane)



Topological location of quasi-periodic orbits

$$H_2 = \rho x_1 y_1 + \frac{\lambda_1}{2}(x_2^2 + y_2^2) + \frac{\lambda_2}{2}(x_3^2 + y_3^2) = h \qquad h > 0 \quad \text{(h: energy level)}$$



Quasi-periodic orbits are characterized by $x_1 = 0$ and $y_1 = 0$. Given a trajectory crossing the equilibrium region with $x_1 = \alpha_1$ and $y_1 = \alpha_2$ it can be redirected towards the quasi-periodic by applying $\Delta x_1 = -\alpha_1$ and $\Delta y_1 = -\alpha_2$

Station-keeping strategy

1. Applying $\Delta x_1 = -\alpha_1$ the trajectory converges asymptotically to the quasi-periodic orbit $(x_1 = 0)$.
2. Including also $\Delta y_1 = -\alpha_2$ the trajectory keeps following the quasi-periodic orbit
3. Converting the two equations above into expressions in position and velocity coordinates (inverse Siegel-Moser transformation) provides the station-keeping linear guidance laws

Station-keeping strategy (impulsive)

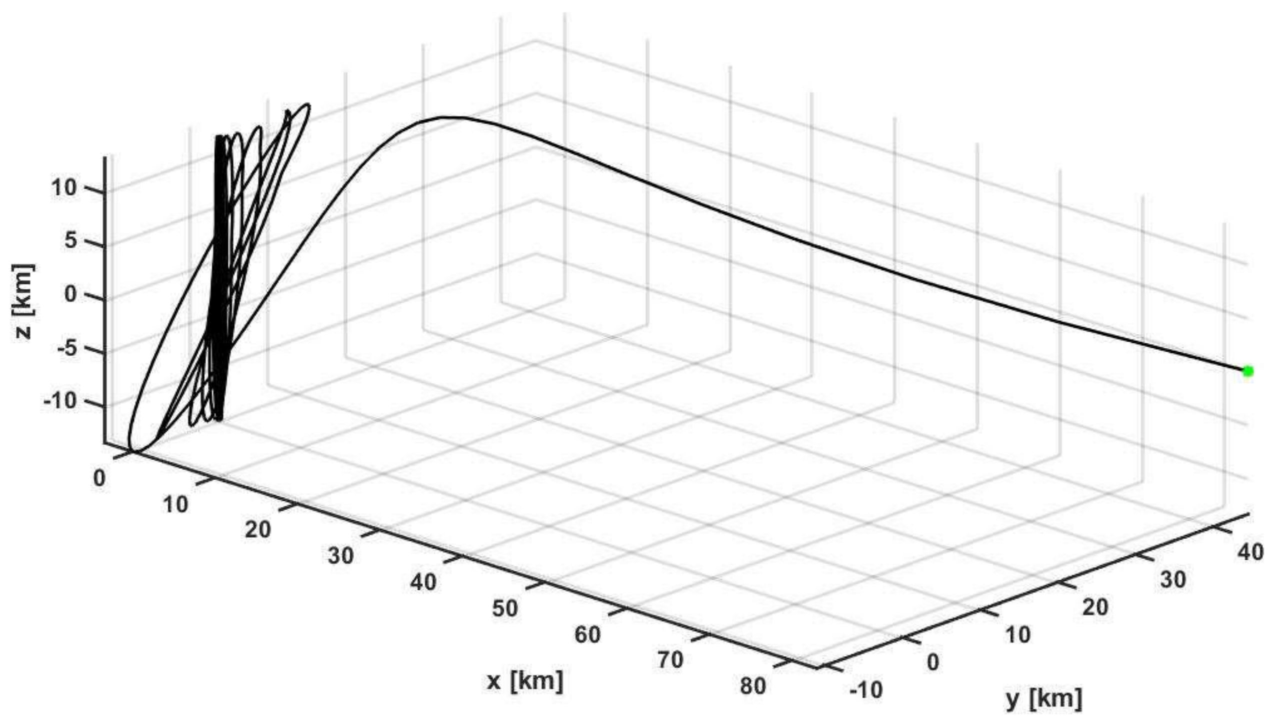
$$\begin{cases} \delta u = -\dfrac{1}{\alpha}[(1+2\gamma^2)x + (1-\gamma^2)y + \alpha u - \alpha\sigma v] \\ \delta v = \dfrac{1}{\alpha\sigma}[(1+2\gamma^2)x + (1-\gamma^2)y + \alpha u - \alpha\sigma v] \end{cases}$$

Station-keeping strategy (continuous)

$$\begin{cases} a_x = -\dfrac{1}{\alpha}[(1+2\gamma^2)x + (1-\gamma^2)y + \alpha u - \alpha\sigma v] \\ a_y = \dfrac{1}{\alpha\sigma}[(1+2\gamma^2)x + (1-\gamma^2)y + \alpha u - \alpha\sigma v] \end{cases}$$

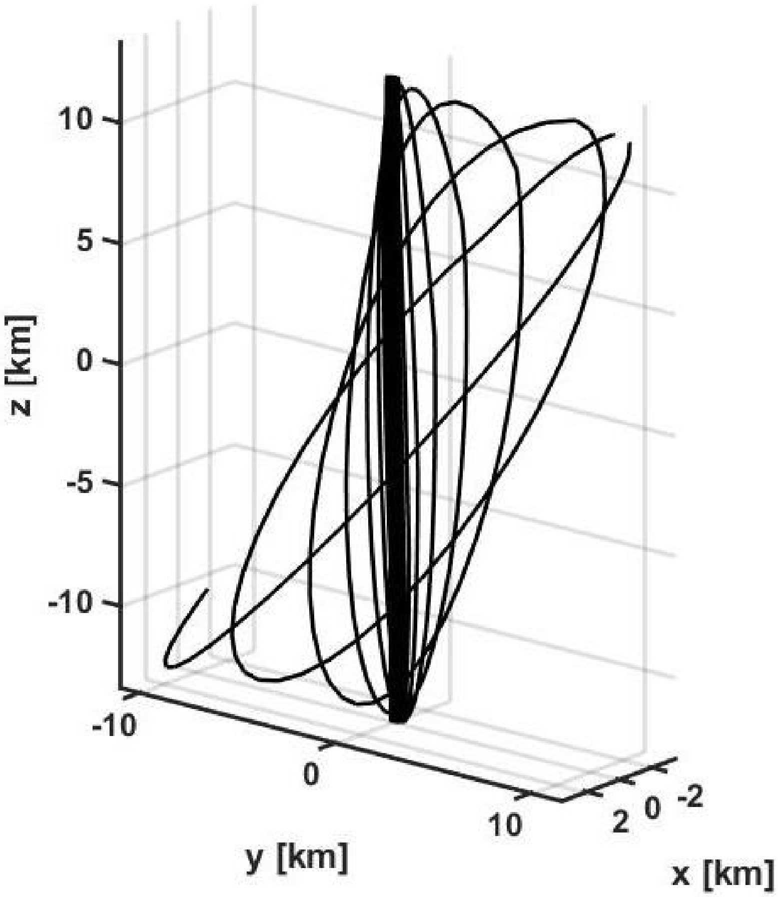
Station-keeping strategy

The station-keeping strategy is verified by means of numerical analysis. A total of 6000 states was selected from initial conditions corresponding to transit trajectories from the Sun to Mars. These initial states are integrated using the nonlinear equations of motion.

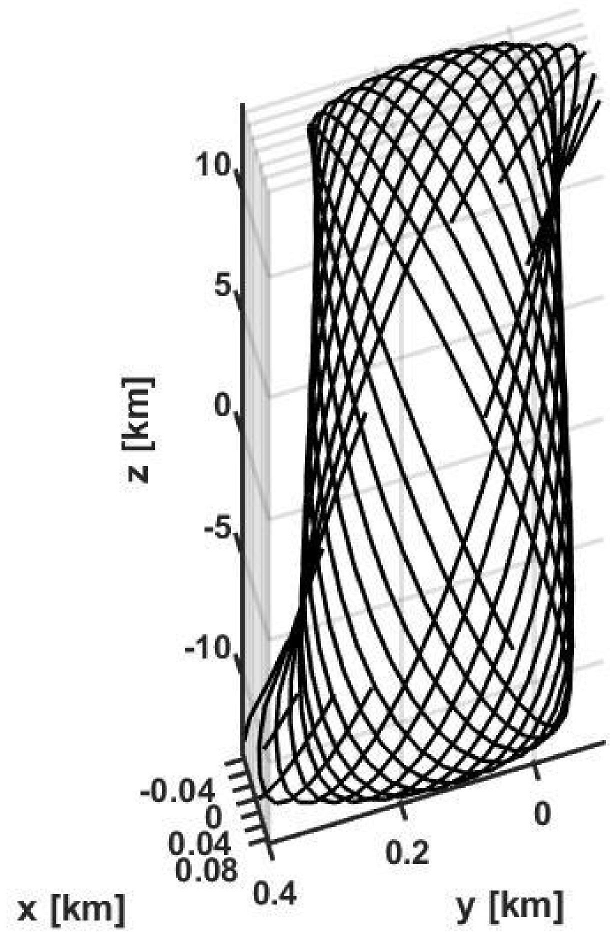


Trajectory converging and keeping a Lissajous orbit about Sun-Mars L_1

Station-keeping strategy

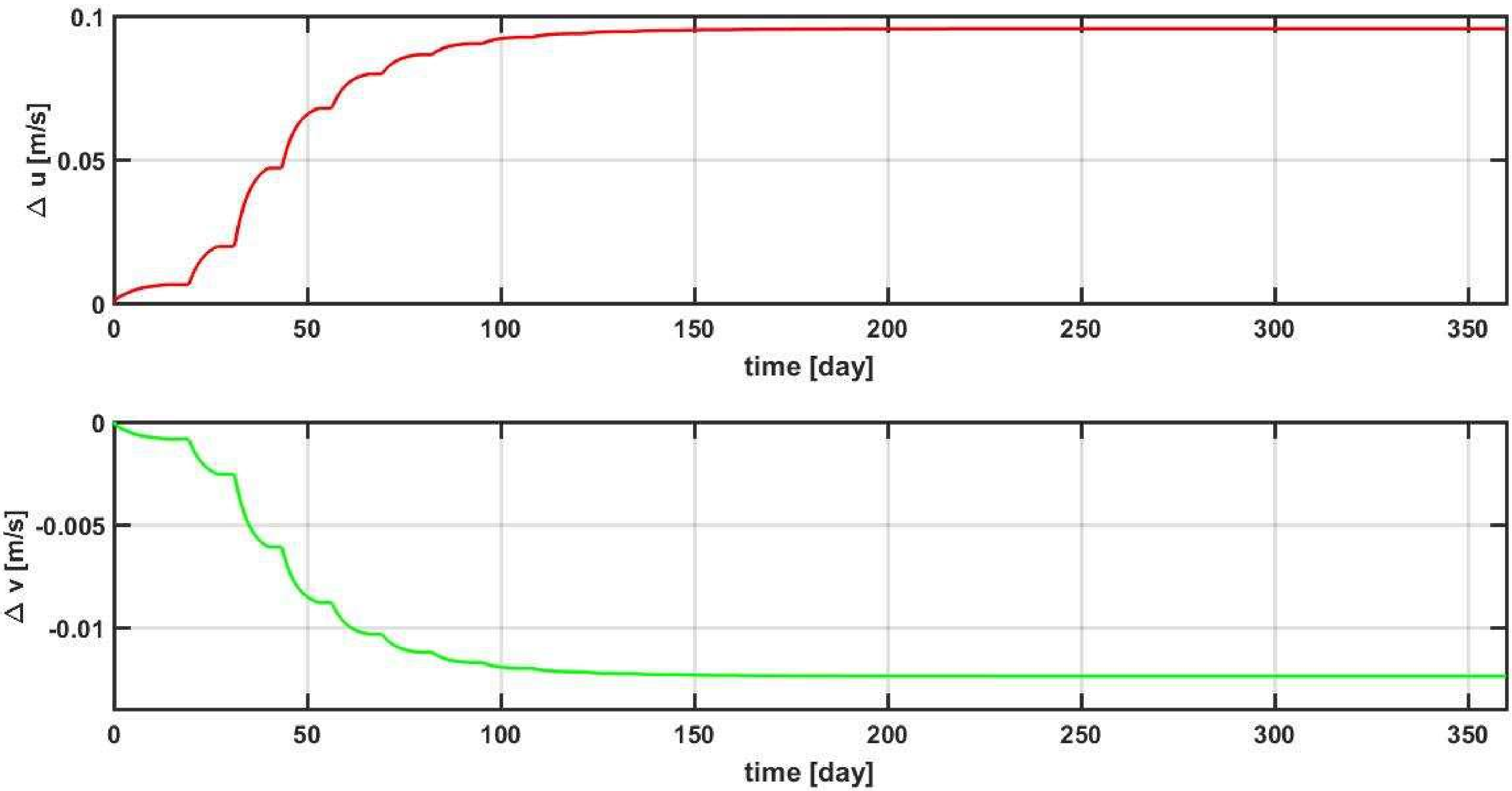


Station-keeping trajectory



Lissajous trajectory

Station-keeping strategy



Cumulative delta-V

Station-keeping strategy

Parameter	Value
Max delta-V [m/sec]	0.1362
Min delta-V [m/sec]	0.0075
Max thrust* [N]	6.491e-5
Min thrust* [N]	3.5771e-6
Max propellant mass** [kg]	0.190

* Considering a 15 kg satellite
** Considering Xe for BIT-1

	RIT 10 EVO [4]	BIT-1 [21]	BIT-3[21]	BGT-X1 [22]	IFM Nano [32]
Max thrust [N]	5e-3	1.85e-4	1.15e-3	0.1	5e-4
Power [W]	50	8	75	4.5	40
Total mass [kg]	1.8	0.75	1.5	1.5	0.87
Volume [U][5]	2	1	2	1	1
Max ΔV [m/sec]	28.8	1.07	6.62	576	2.88

Commercial thrusters properties

Conclusions

- A compact topological description of quasi-periodic orbits for the Sun-Mars ER3BP was provided
- Based on this representation, conditions driving a transit trajectory to the quasi-periodic orbit and station keeping were determined
- The mentioned conditions were converted into guidance laws and verified by means of numerical analysis
- The results indicate that station-keeping can be achieved at limited delta-V, thrust and propellant mass budget. The strategy is compatible with current technology.

Thanks for your attention! Questions?

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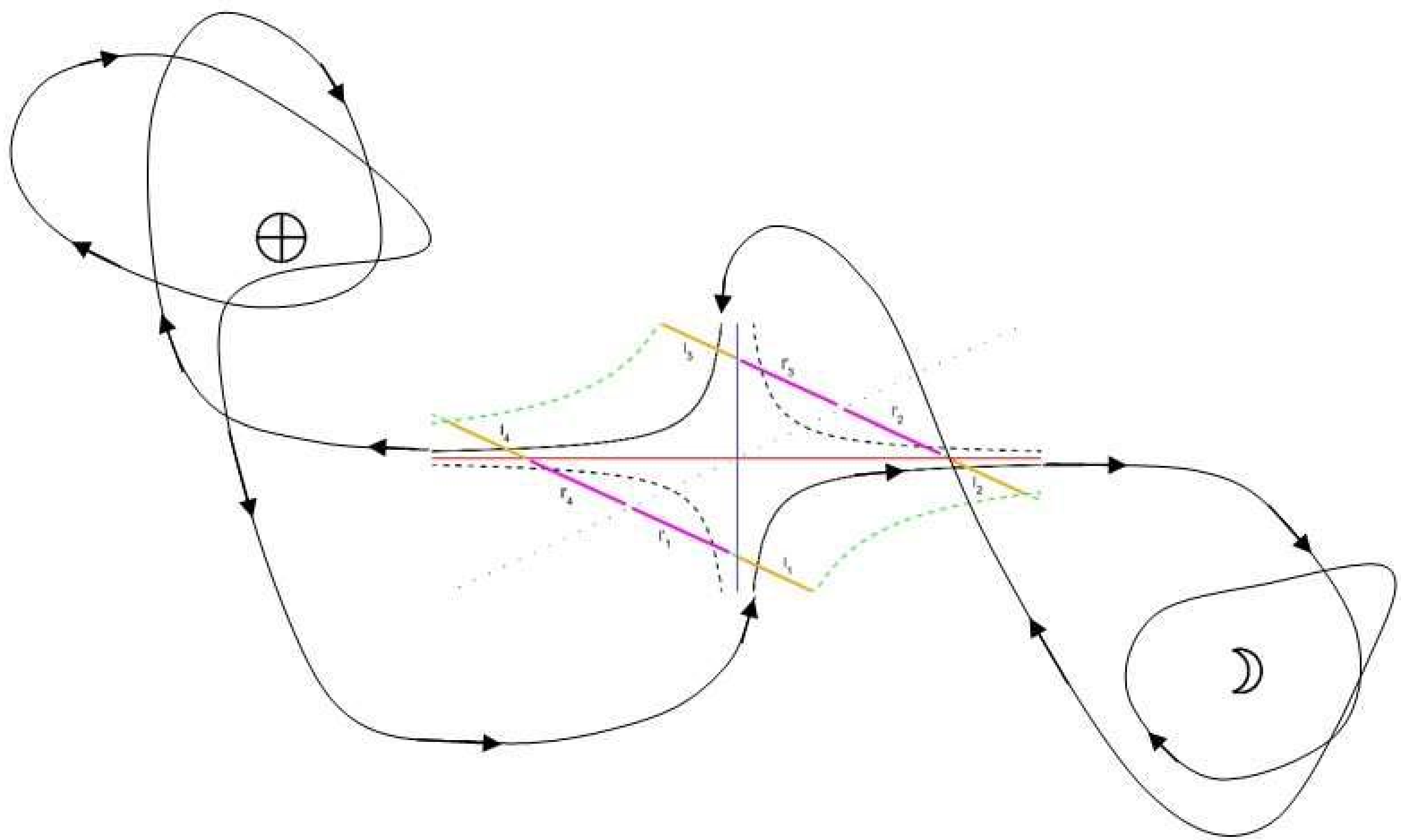
1. Solar system exploration using CubeSats

1. Proposed solution: Ballistic captures in the CR3BP

Transit trajectories can be investigated onto the [x1,y1] plane



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