MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY

Methods for accurate ballistics calculations for multi-satellite constellations

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Motivation

Global constellations

The need for real-time computing

The increase in the amount of space debris

⇨ The need of **fast** and **accurate** methods

State of Art: Mathematical model

Newton's Law

$$\ddot{r} = \sum F_i(t, r, \dot{r}) = \nabla V + g_{moon} + g_{sun} + \frac{F_{atm}}{m}$$

Gravity and Tides

$$V(r, \varphi, \lambda) = \frac{GM}{r}\left[1 + \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin\varphi)(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda))\right]$$

$$F = m\nabla V$$

Sun

$$\vec{\mathrm{F}} = \frac{-\vec{R}}{|R|}\Phi\frac{\mathrm{Ak}\theta}{c}\frac{r_s^2}{R^2}$$

\+ Earth rotation
Precession nutation

Moon

$$\vec{F}_i = -\frac{fM_S m}{(R)^3}\vec{R} - \frac{fM_M m}{(\vec{R}_M)^3}(\vec{R}_M)$$

Atmosphere

$$F_{atm} = S\rho_V u^2$$

One-step methods

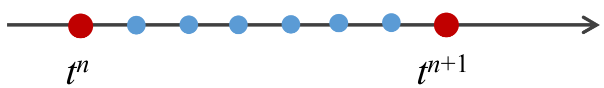
$$\dot{u} = f(t,u)$$

$$\int_{t^n}^{t^{n+1}} du = \int_{t^n}^{t^{n+1}} f dt \sim \tau \sum_{i=1}^{r} \gamma_i f_i$$

$$u^{n+1} - u^n \sim \tau \sum_{i=1}^{r} \gamma_i f_i$$

$$\frac{u^{n+1} - u^n}{\tau} = \sum_{i=1}^{r} \gamma_i k_i \qquad u^{n+1} = u^n + \tau \sum_{i=1}^{r} \gamma_i k_i$$

$$k_i = f(t_n + \alpha_i \tau, u^n + \tau \sum_{j=1}^{r} \beta_{ij} k_j)$$



α_1	β_{11}	...	...	β_{1r}
...	...	...		
...	...		...	
α_r	β_{r1}			β_{rr}
	γ_1	...	...	γ_r

Embedded Methods: Dorman-Prince

Dorman-Prince 5(6) , 7(8) ~ 1981

0	0	…	…	0
α_2	β_{r2}	…		
…	…		…	
α_r	β_{r1}			0
	γ_{11}	…	…	γ_{1r}
	γ_{21}	…	…	γ_{2r}

$$u_1^{n+1} = u_1^n + \tau \sum_{i=1}^{r} \gamma_{1i} k_{1i}$$

$$u_2^{n+1} = u_2^n + \tau \sum_{i=1}^{r} \gamma_{2i} k_{2i}$$

$$\Delta^n = \|u_1^n - u_2^n\|$$

$$\tau$$

Advantages:

- Easy to use
- Suitable for highly elliptical orbits

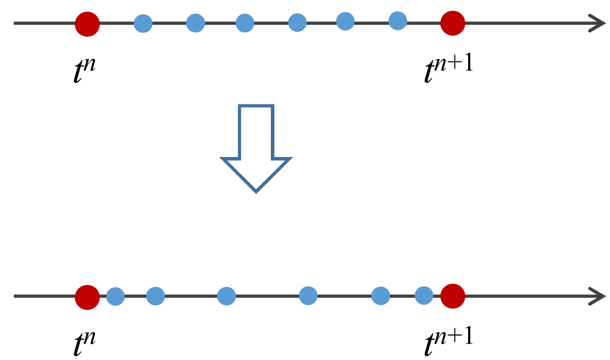
Disadvantages:

- Time step limits
- Limitations on Computing Speed

Gauss Quadratures

$$\int_{t^n}^{t^{n+1}} du = \int_{t^n}^{t^{n+1}} f(t,u)dt$$

$$u^{n+1} - u^n = \tau \sum C_i f_i$$



α_1	β_{11}	…	…	β_{1r}
…	…	…		
…	…		…	
α_r	β_{r1}			β_{rr}
	γ_1	…	…	γ_r

Everhart (~1980)

$$u^{n+1} = u^{n+1} + \tau \sum_{j=1}^{s} \frac{a_j \tau^j}{j}$$

$$\bar{h} = h\left(\frac{s}{h}\frac{e_{toll}}{\|a_s\|}\right)^{1/s}$$

Advantages :

- Automatic time step
- Convergence in 1-2 iterations

Disadvantages :

- It is necessary to solve a nonlinear system of equations

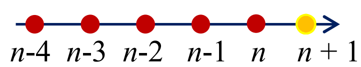
Multistep Methods (Adams)

Explicit

$$u_1^{n+1} = u_1^n + \tau \sum_{i=1}^{k} \alpha_i f^{n+1-i}$$

Implicit

$$u_1^{n+1} = u_1^n + \tau \sum_{i=0}^{k} \alpha_i f^{n+1-i}$$



Gauss-Jackson (1929)

Explicit Predictor + Implicit Corrector

Advantages :

- Nonlinear iterations for only one equation
- One of the fastest methods

Disadvantages :

- Time step fixed
- Inconvenient for parallel implementation
- A-stability =?

$$\int_{-1}^{1} f(x)W(x)\,\mathrm{d}x = \sum_{j=1}^{M} w_j f(\tau_j),$$

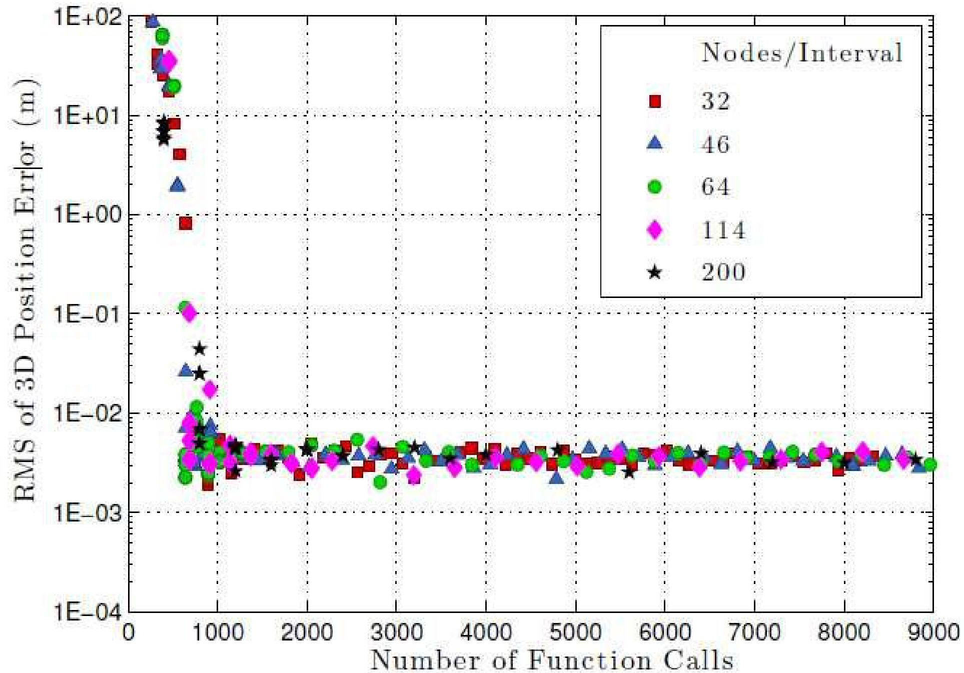
вес

$$\left|\int_{-1}^{1} e^{2ictx} W(t)\mathrm{d}t - \sum_{j=1}^{M} w_j e^{2ic\tau_j x}\right| < \epsilon^2, \quad x, \tau_j \in [-1, 1]$$

For BLC-IRK $W(t) = 1$

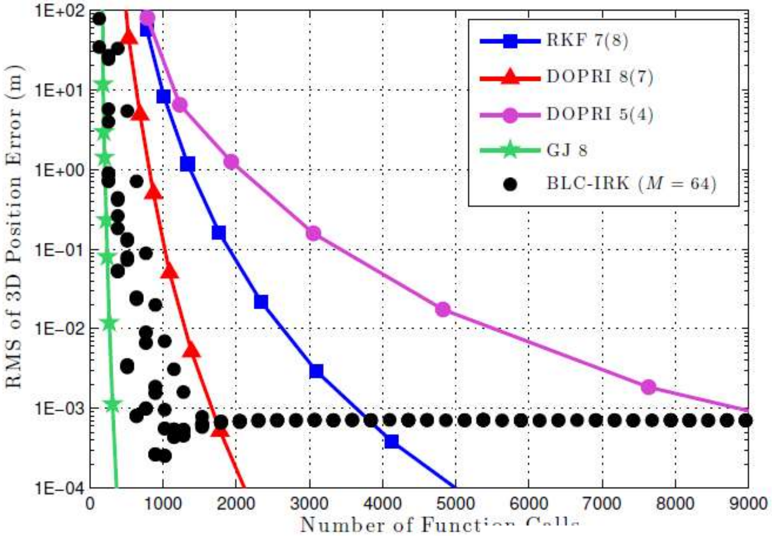
At the moment, 5 method parameters must be determined manually:

- Accuracy **ε**
- Number of nodes M
- Time step h
- The number of iterations with low accuracy N_1
- The number of iterations with high precision N_2

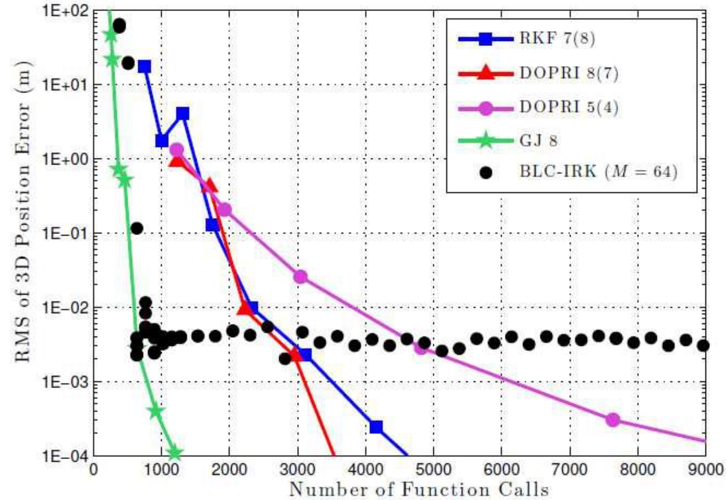


Main property: The function value in each node can be determined independently of the others.

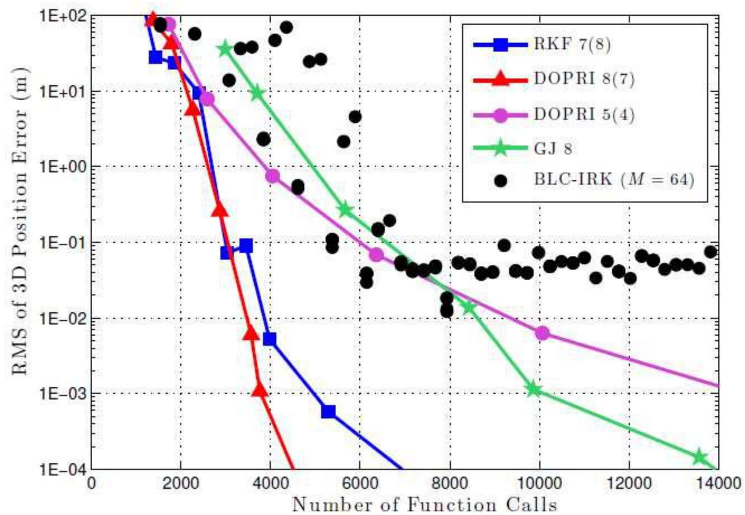
[B.K. Bradly Numerical Algorithms for Precise and Efficient Orbit Propagation and Positing, 2015]



GEO



LEO



Orbit Molnia

[B.K. Bradly Numerical Algorithms for Precise and Efficient Orbit Propagation and Positing, 2015]

Problem of determining the right-hand side of the system

$$\ddot{r} = \sum F_i\,(t, r, \dot{r}) = F_{\text{Earth}} + g_{\text{Moon}} + g_{\text{Sun}} + F_{\text{Atmo}} + F_{\text{Sun pressure}}$$

$$V(r, \varphi, \lambda) = \frac{GM}{r}\left[1 + \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin\varphi)\left(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\right)\right]$$

$$P_n^m(\sin\varphi) = \cos^m\varphi\,\frac{d^m}{d(\sin\varphi)^m}P_n(\sin\varphi)$$

$$\sin x = \sum_{k=0}^{K}(-1)^k\frac{x^{2k+1}}{(2k+1)!}$$

Up to 4 embedded loops at each time step!

Polynomial representation of the spherical part

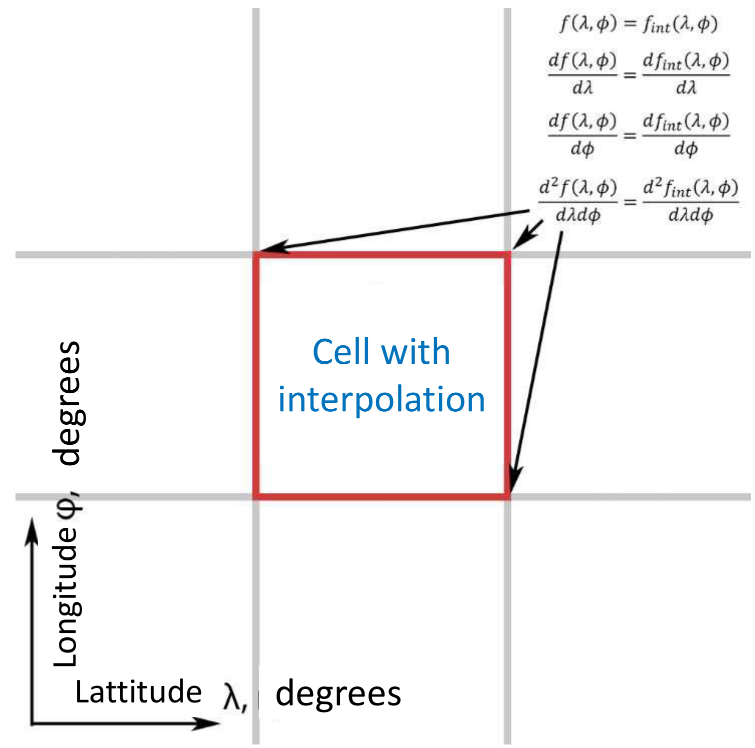
Interpolation by two-dimensional cubic splines

$$\frac{M_4}{384} h_{\max}^4 \leq 10^{-8}$$

$$h \sim 1..5^{o}$$

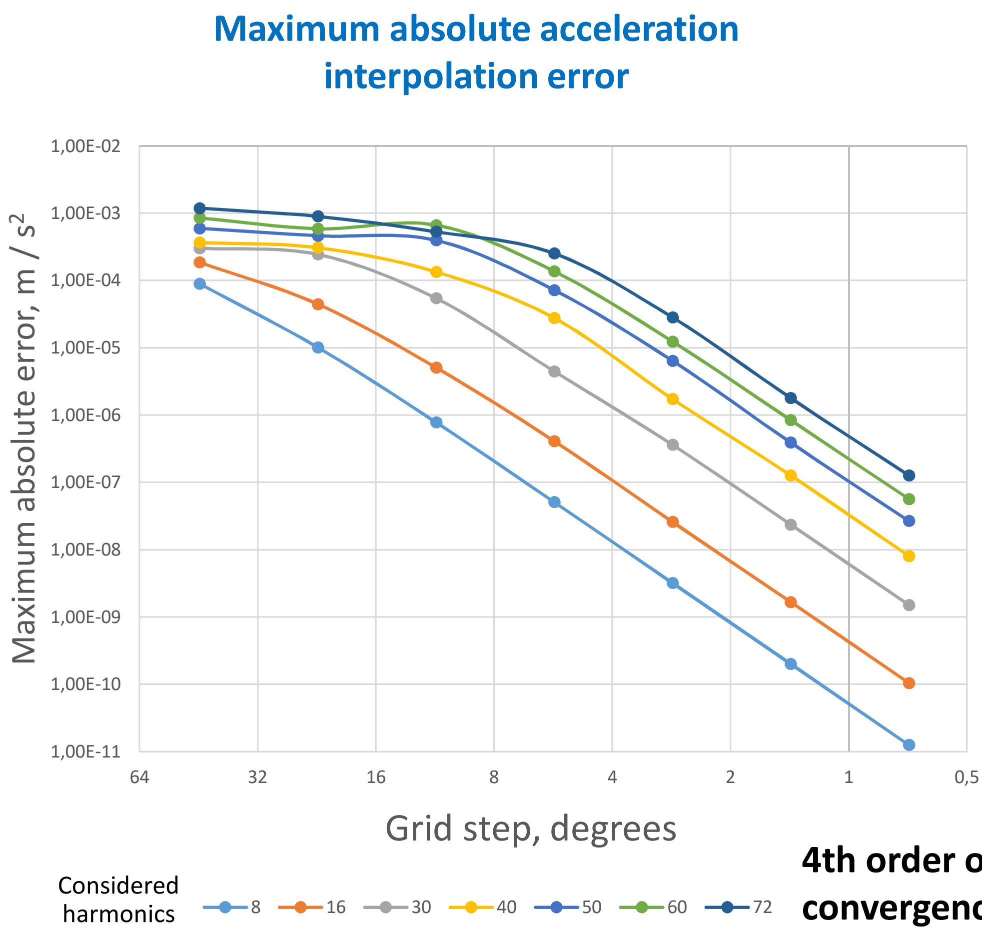
$$U = \frac{fM}{r_0}\left\{\sum_{n=2}^{\infty}\left(\frac{r_0}{r}\right)^{n+1}\sum A_{kl}\varphi^k\lambda^l\right\}$$

2 embedded loops at each time step

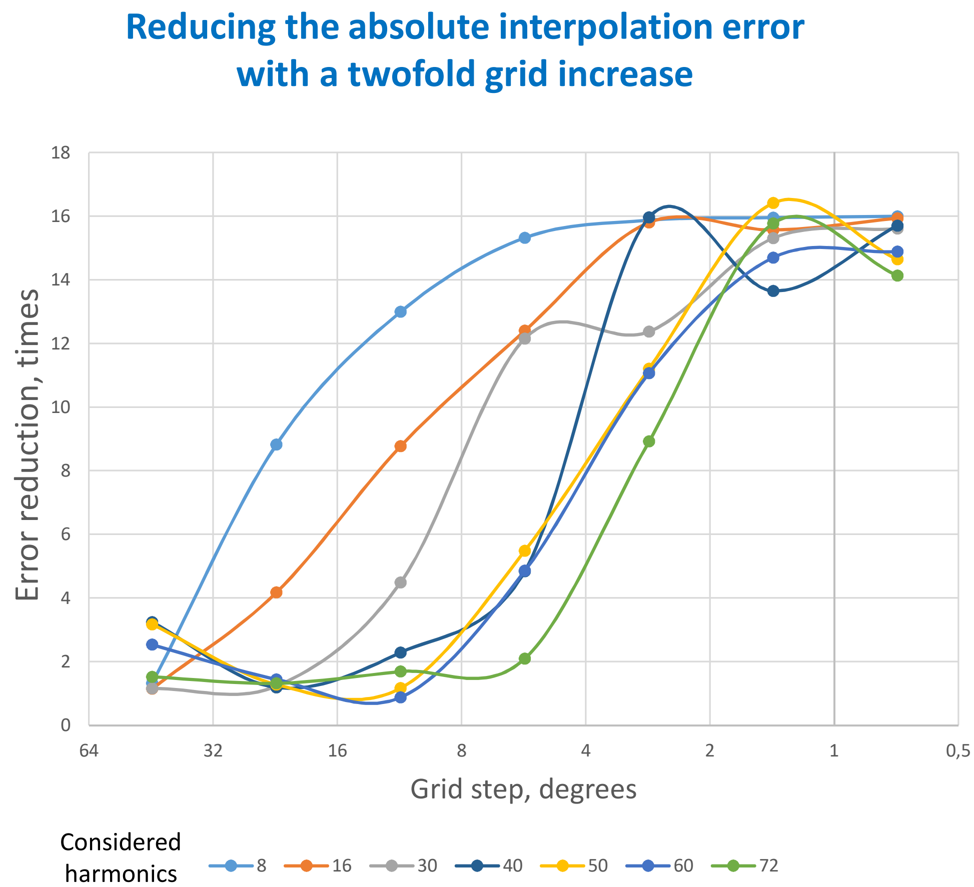


Interpolation of a function on a two-dimensional grid.

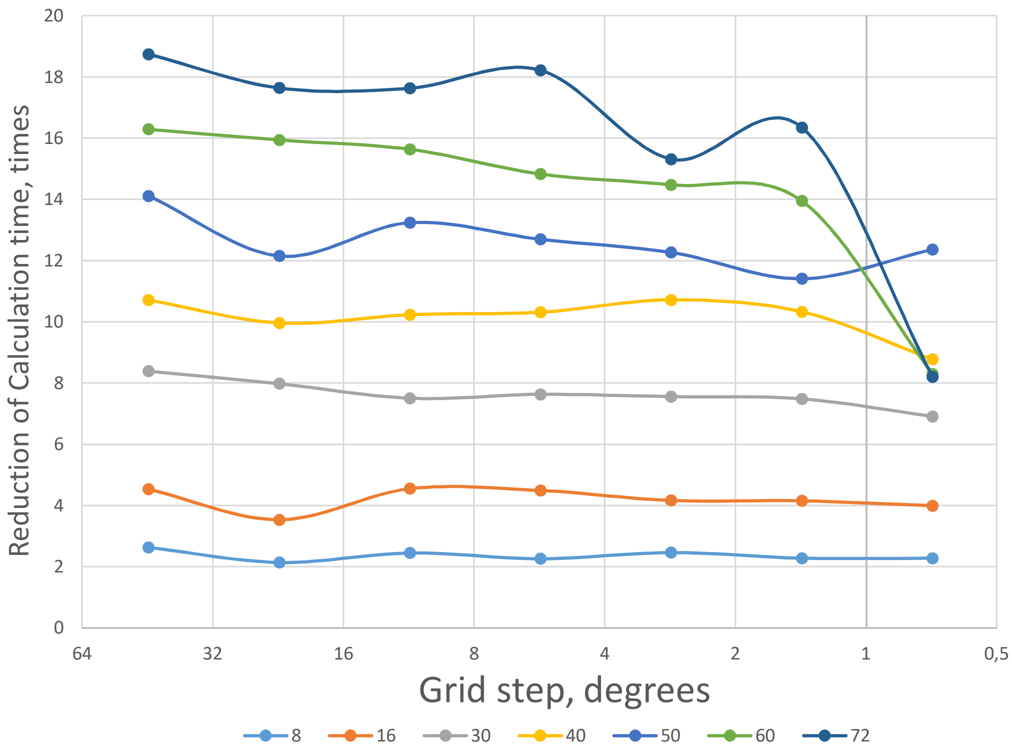
Interpolation error



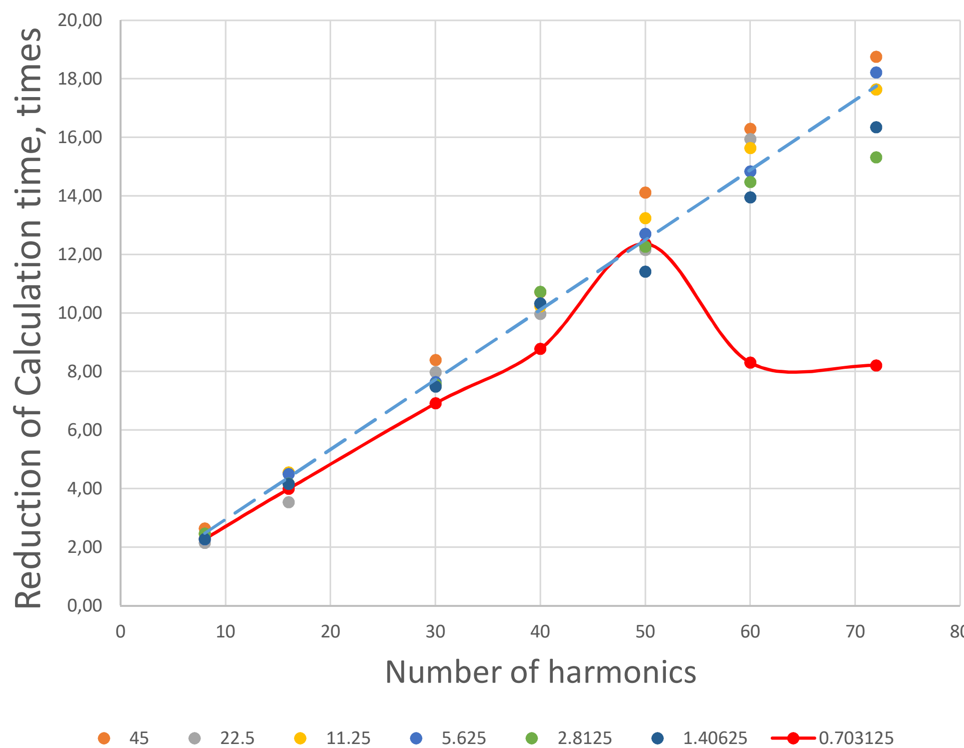
4th order of convergence



Reduced calculation time when interpolating acceleration terms for one point



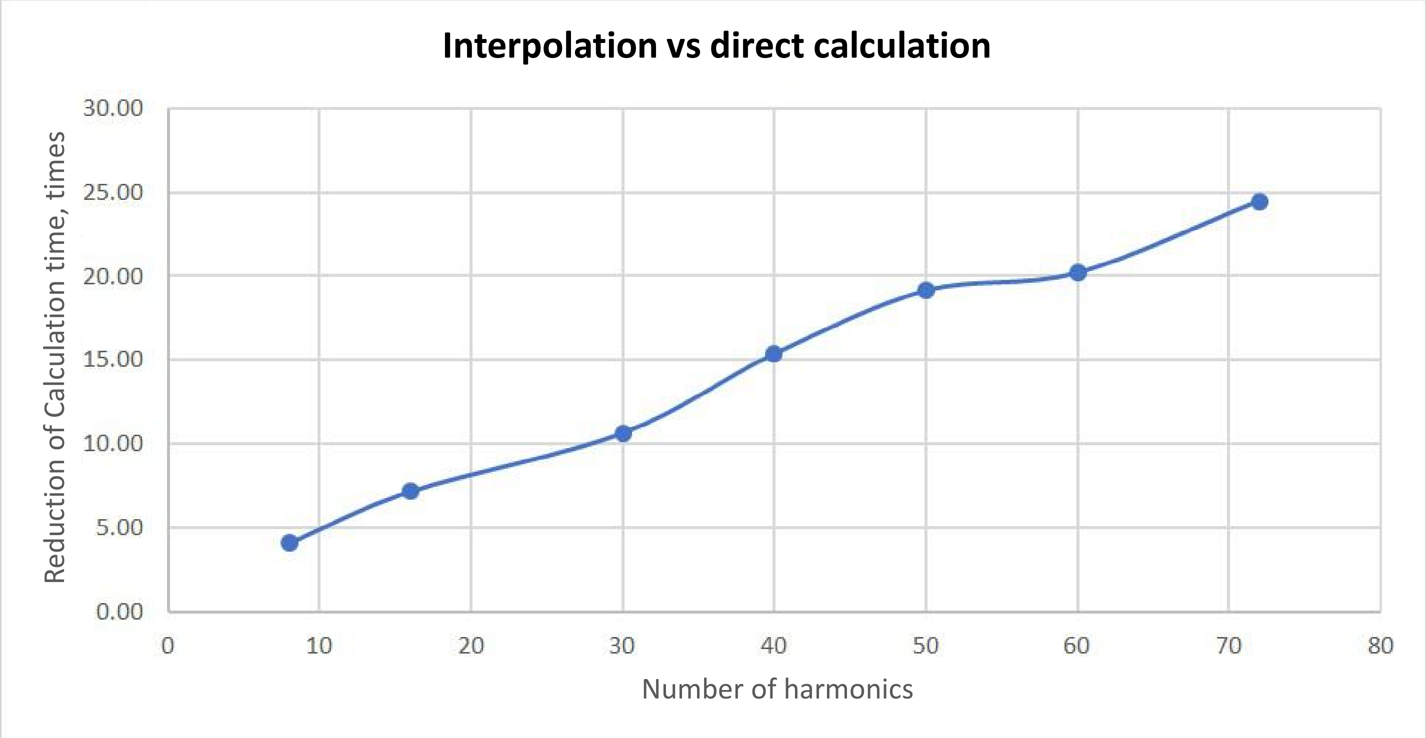
Reduced calculation time when interpolating acceleration terms



Results

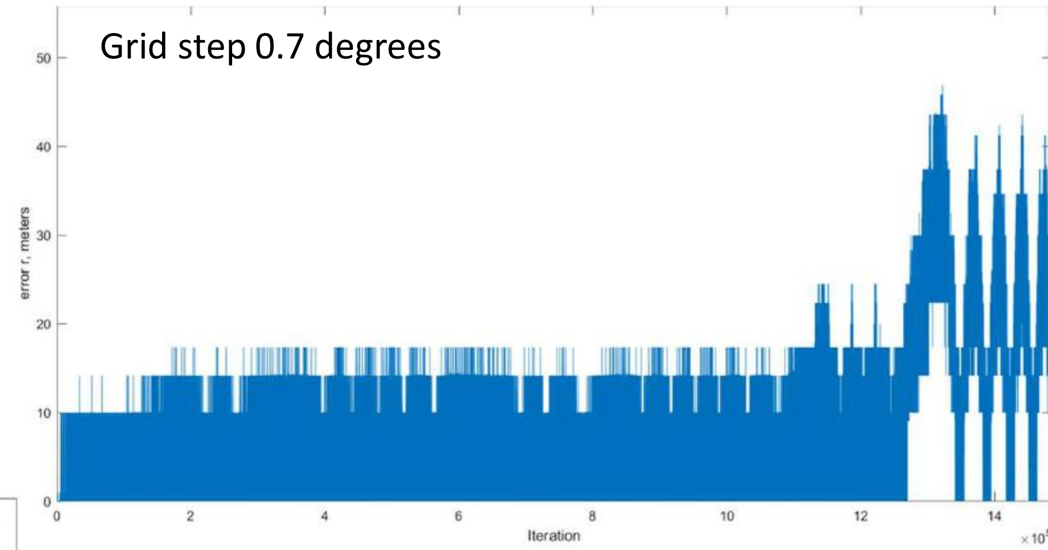
Calculation of the dynamics of one spacecraft

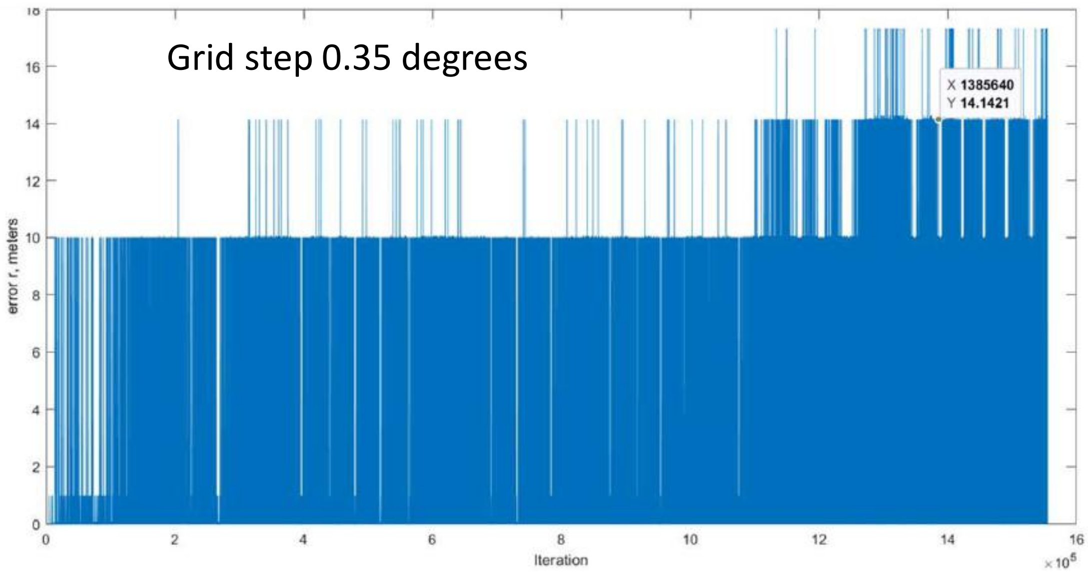
Time period, sec	2.59E+06	~ 1 month					
Time step, sec	30						
Grid step, degrees	0.703125						
Number of harmnics	8	16	30	40	50	60	72
Direct computation, sec	1.24	5.68	11.74	20.06	30.59	43.50	62.27
Interpolation, sec	0.30	0.79	1.10	1.31	1.60	2.16	2.55
Time gain, times	4.09	7.17	10.65	15.35	19.12	20.18	24.45



Error determining the position of the spacecraft

The study of the error in determining the position of the spacecraft in orbit for **6 months**





CONCLUSION

- A method for reducing the computational complexity of ballistic calculations by simplifying the right side of the system of equations is proposed

- Using bicubic splines, an interpolation of the gravitational potential was constructed with errors not exceeding the errors in determining the expansion coefficients of the gravitational potential

- The technique has demonstrated its efficiency and the ability to accelerate ballistics calculations up to 25 times in the case of using a large number of harmonics

Thank you for your attention!