SAPIENZA
UNIVERSITÀ DI ROMA

since 1926 SIA - Scuola di Ingegneria Aerospaziale

Laboratorio di Guida e Navigazione

Collision Avoidance for CubeSats in Formation Flying

Karthick Dharmarajan - Marco Sabatini - Giovanni B. Palmerini

Introduction

Currently, there are more than 18000 artificial satellites in space and more than 35000 objects larger than 10cm with another million objects large enough to cause serious damage to the satellites. These objects include non functional satellites, space debris and even meteoroids orbiting the earth.

A recent report by Amazon on its planned 3236 Kuiper system satellites shows that even if 1 in 10 satellites fails in orbit there is a 12 percent chance that it will collide with a space debris bigger than 12 cm. This startling report calls for an extended and improved tracking and surveillance action for the operational satellites.

There is a clear, urgent need to exploit **collision avoidance manoeuvers** (and indeed to investigate their nature in order to optimize them).

Notwithstanding their limitations in thrust available onboard, such a need is important for **Cubesats** too: (i) a collision would generate relevant problems, and (ii) their missions are becoming increasingly complex and valuable, so care should be devoted to their own safety

Introduction (2)

The problem is even larger when considering formations, with 2 possible occurrences for collision avoidance:

1. when a foreign object enters the formation.

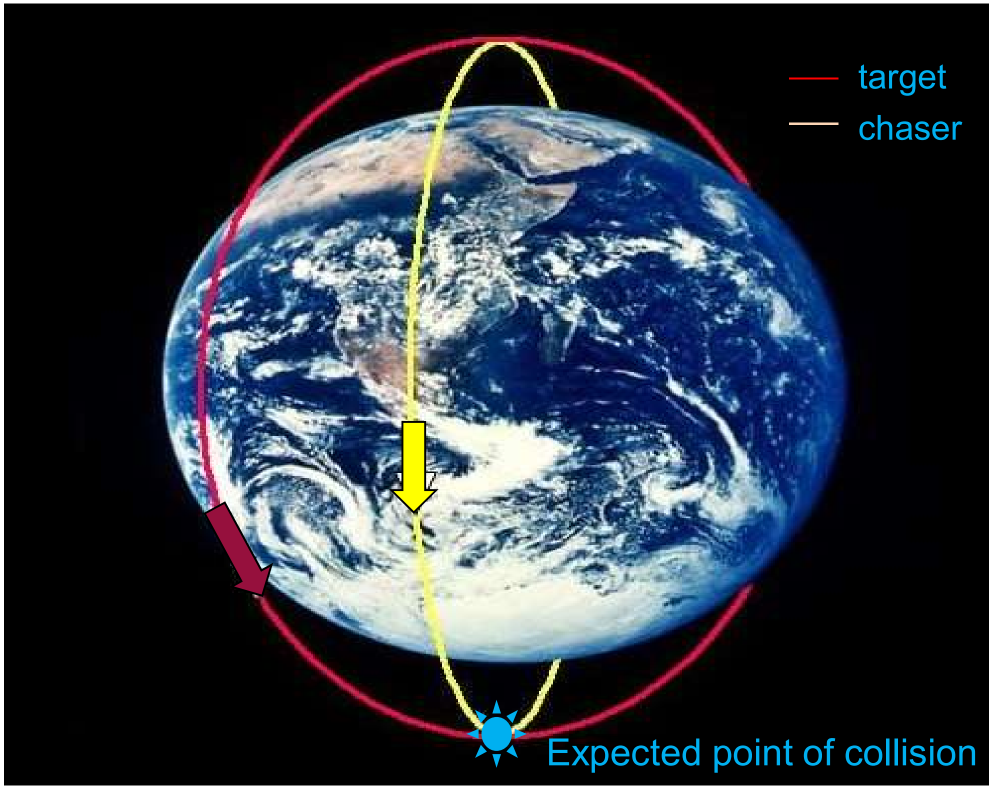
2. one of the satellite in formation drifts closer to another satellite and eventually collide as a result of unaccounted perturbations, imperfect control or maybe an imperfect satellite launch resulting in an undesirable trajectory.

It is possible to investigate the problem via a number of numerical techniques like Particle Swarm Optimization and Genetic Algorithms

However, to get a better idea on the nature of the problem a more analyical approach is useful. We will start from the approaches by Slater et al.* and by Bombardelli and Hernando/Ayuso #, built on impulsive manoeuvers

* Slater, Byram, Williams "Collision Avoidance for Satellites in Formation Flight", *JDGC,* 2006
# Bombardelli, Hernando-Ayuso, "Optimal Impulsive Collision Avoidance in Low Earth Orbit ", *JDGC*, 2015

A sketch of the problem



Target : the satellite not performing any manoeuver

Chaser: the satellite performing the anti-collision manoeuver

Manoeuver anticipation angle/time: difference in true anomaly (or the corresponding Δt) between avoidance manoeuver execution point and the collision point

Direct/non direct impact

- The final position of the spacecraft after a collision avoidance manoeuver can be represented as

$$\vec{r} = \overrightarrow{r_E} + M\overrightarrow{\Delta V}$$

- Where R is the expected miss distance after the collision avoidance manoeuver and R_E is the expected miss distance without manoeuver

$$M = RKD$$

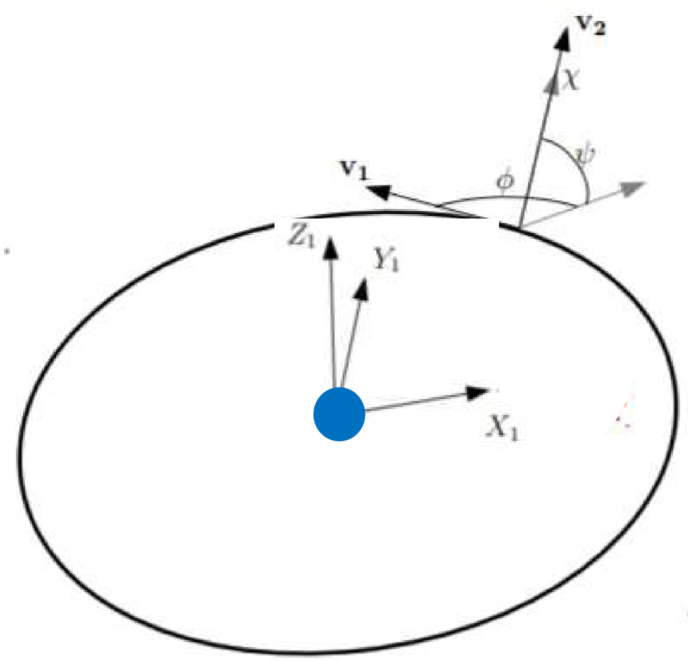
- M is a matrix given by the product of matrix of Rotation, kinematics and Dynamics and ΔV the applied Impulse:

- The direct impact is a special case of collision impacts where the predicted miss distance R_E=0. However this is quite rare and most collision avoidance manoeuvers are usually performed for the non-direct impacts having $R_E \neq 0$.

Collision Avoidance Manoeuvre Example

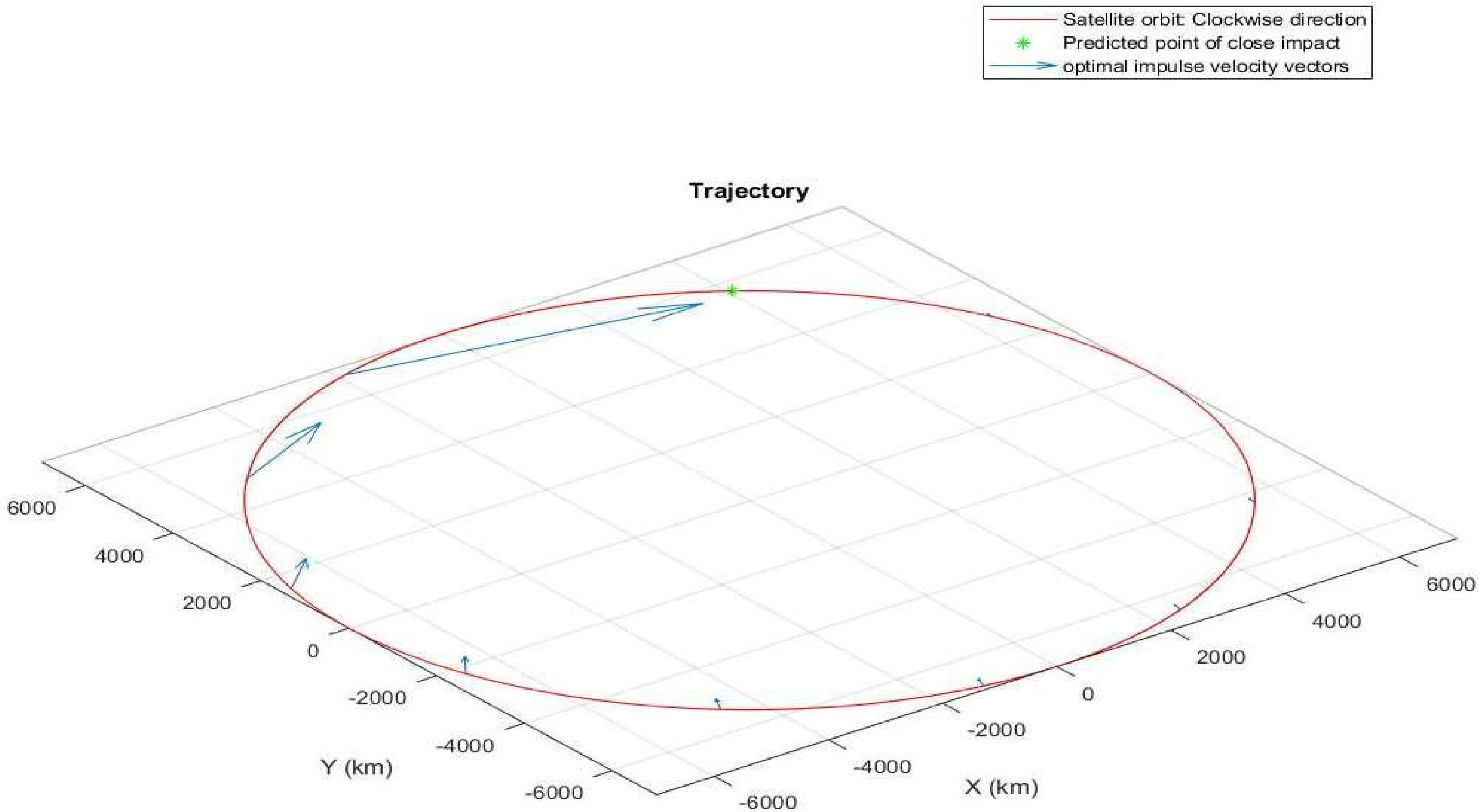
The Iridium-Cosmos collision is taken as a reference for the collision avoidance technique in the B-plane reference frame.

Semiaxis(km)	Eccentricity	ϕ(deg)	Ψ(deg)	Θ_c(deg)	X
7155.8	$2 * 10^{-4}$	180	77.5	-16.85	1



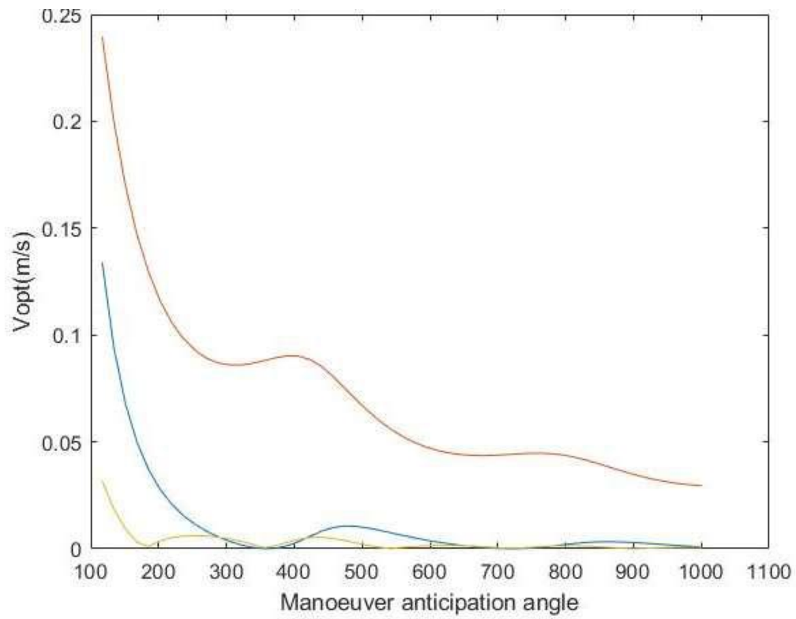
ϕ and ψ are the angles given by rotation from $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ and X is the ratio of the magnitudes of the velocities.

Findings



Closer to the collision, higher the required Delta ΔV

Non Direct Impact – Delta V optimization



The initial conditions are given by,

$$minimize\ \Delta V(r^2) = \Delta V^T \Delta V$$

$$subject\ to\ (r_e + M\Delta V)^T Q (r_e + M\Delta V) - d^2$$

$$\frac{dL}{d\Delta V} = -\lambda (I + \lambda A)^{\dagger} b$$

- The cost of the manouever decreases as the the time betwen manoeuver and collision increases. *Performing manoeuver earlier is not always an advantage.*
- Also note that for the increased anticipation time the tangential component of the optimal velocity impulse becomes more prominent

Solution

Deriving the eigenvalue problem gives us the following equation in lambda

$$F(\lambda) = \frac{(S_1^T b)^2}{c} \frac{\lambda(2 + \lambda_1 \lambda)}{(1 + \lambda_1 \lambda)^2} + \frac{(S_2^T b)^2}{c} \frac{\lambda(2 + \lambda_2 \lambda)}{(1 + \lambda_2 \lambda)^2} - 1 = 0$$

S_1 and S_2 are the eigenvectors of the matrix A

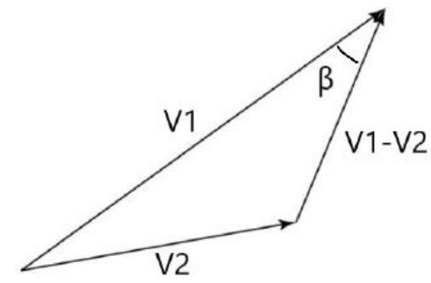
The non linear equation can be solved by using Newton Raphson approach. The initial guess for λ is assumed by first keeping S_1=0 and then S_2=0 , with the max value providing the initial guess:

$$\lambda_0 = \max\left(\frac{1}{\lambda_1}\left(-1 + \frac{1}{\sqrt{1 - \frac{\lambda_1 c}{(S_1^T b)^2}}}\right), \frac{1}{\lambda_2}\left(-1 + \frac{1}{\sqrt{1 - \frac{\lambda_2 c}{(S_2^T b)^2}}}\right)\right)$$

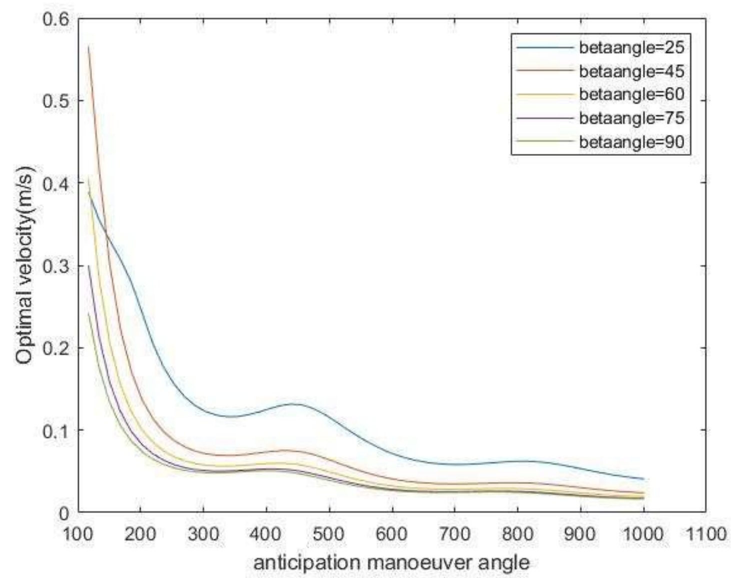
The solution from Newton Raphson (i.e. λ_{opt}) can be substituted in the above eigenvalue problem to obtain the optimal velocity.

The β angle

β angle : the angle between $\overrightarrow{V_1}$ and $\overrightarrow{V_1}$-$\overrightarrow{V_2}$ (1 chaser, 2 target) approaching 90°in formation flying (velocity almost equal in value and direction)



From previous results it derives that as the velocity vectors are equal in magnitude and similar in direction the optimal velocity required is very low. Considering the conditions given for B-plane the β angle is given by



$$cos\beta = \frac{1 - Xcos\psi cos\phi}{\sqrt{1 - 2Xcos\psi cos\phi + X^2}}$$

Collision Avoidance in Formation Flying

Initial conditions given by the Hills or HCW equations to define the formation dynamics. All perturbations are neglected and the leader orbit is assumed as circular.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 0 & 2\omega & 0 \\ 0 & 0 & 0 & -2\omega & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 & 0 & 0 \end{bmatrix}$$

Collision avoidance manouever (Formation Flying)

The fundamental equation of an impulse manouever is given by,

$$\begin{bmatrix} r \\ V \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} r0 \\ V0 + \Delta V \end{bmatrix}$$

$$\phi_{11} r0 + \phi_{12} V0 \neq 0 = r_e$$

$$r = r_e + \phi_{12} \Delta \mathrm{V}$$

$$V = \phi_{21} r0 + \phi_{22} V0 + \phi_{22} \Delta \mathrm{V}$$

The optimization problem becomes, $minimize\ \Delta V(r^2)$

$$subject\ to\ f(r^2) = r^2 - d^2$$

Solving this problem gives an Eigenvalue problem given by,

$$\frac{dL}{d\Delta V} = -\lambda (I + \lambda A)^{\dagger} b$$

similar to what we got for the non-direct case in the B-Plane. Solving it will provide the optimal velocity required for the manouever.

A better solution

The collision avoidance manouever presented by the Bombardelli approach is generally sufficient for most of the formation flying manoeuver cases. However

- That approach works on the condition that in a reference frame defined by the collision geometry the propagated orbit is a straight line and indeed a tangent is created as the collision avoidance solution. This approximation is valid for most collision scenarios. But in an LVLH plane even for small propagation times the approximation is not valid.
- It is better to use an iterative approach to propagate multiple tangents and hence define a curve that satisfies the solution.

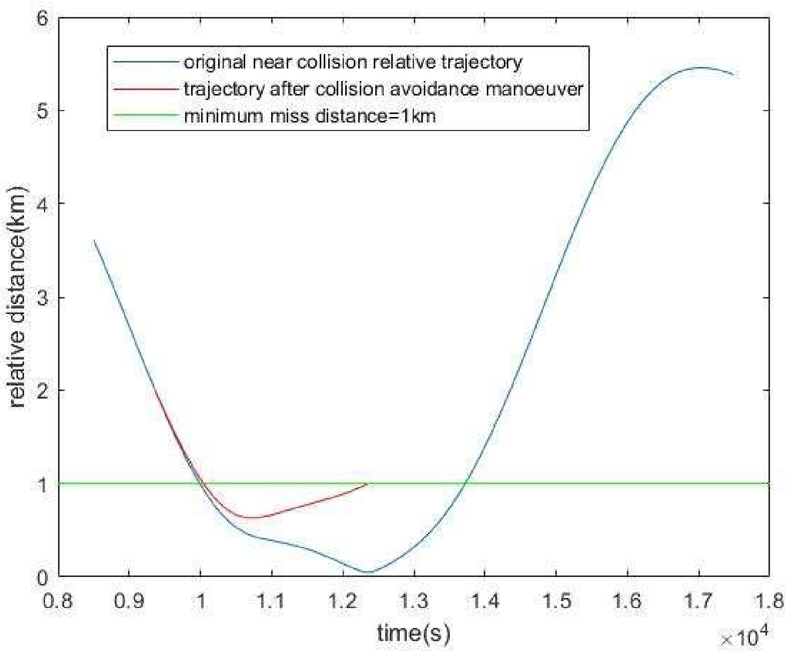
Setting the initial condition of the manoeuver to

required miss distance > projected miss distance

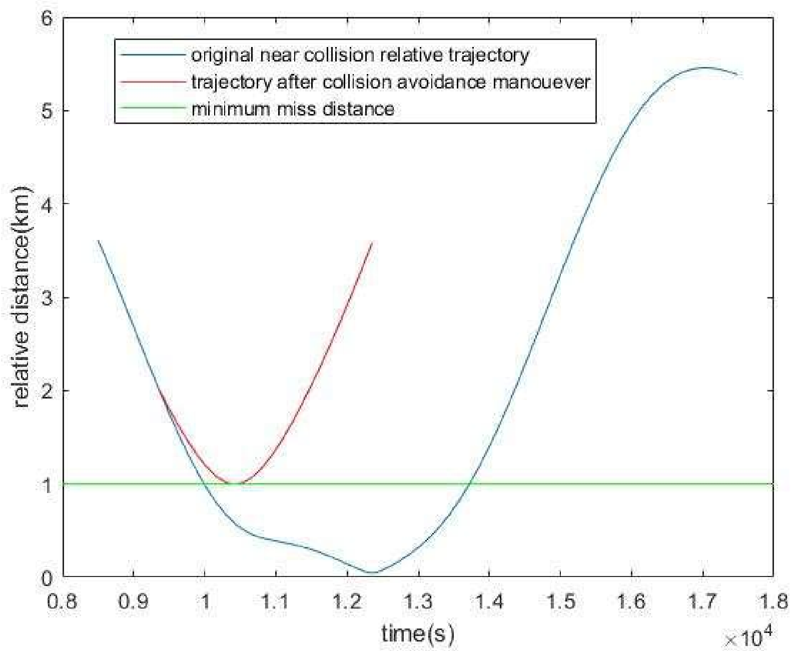
we can iterate on the optimal velocity calculation in the previous slides by propagating the orbits repeatedly by adding the optimal deltav from the previous iterations to satisfy the miss conditions.

Collision avoidance manoeuver (Formation Flying)

ΔV Bombardelli=0.0899927 m/s
ΔV iteration approach=0.334925 m/s

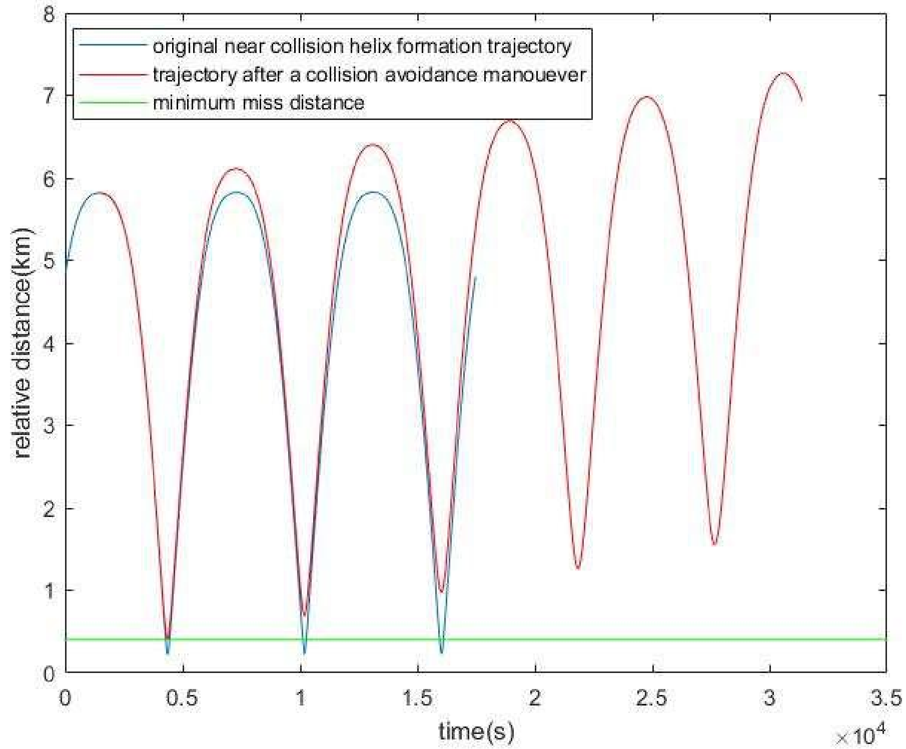


a)Collision manoeuvre with Bombardelli approach



b)Collision manoeuvre after iteration

Helix relative trajectories



Required miss distance=0.4 km
Manoeuver anticipation time=3000s
Vopt collision =0.0354 m/s

The helix trajectory observed in TANDEM-X formation is a cartwheel formation, characterized by difference in eccentricity and a difference in argument of latitude. While projecting the trajectory from equator there is a significant collision risk as can be seen above and a collision manouever maybe required to prevent a collisions. As can be seen above the manouever successfully reduced the risk of collision

Collision Avoidance in Formation Flying

Formation flying poses a special and additional requirement:

There is a need to bring back the satellite to the original formation or at least a formation where the risk of further collisions is considerably less.

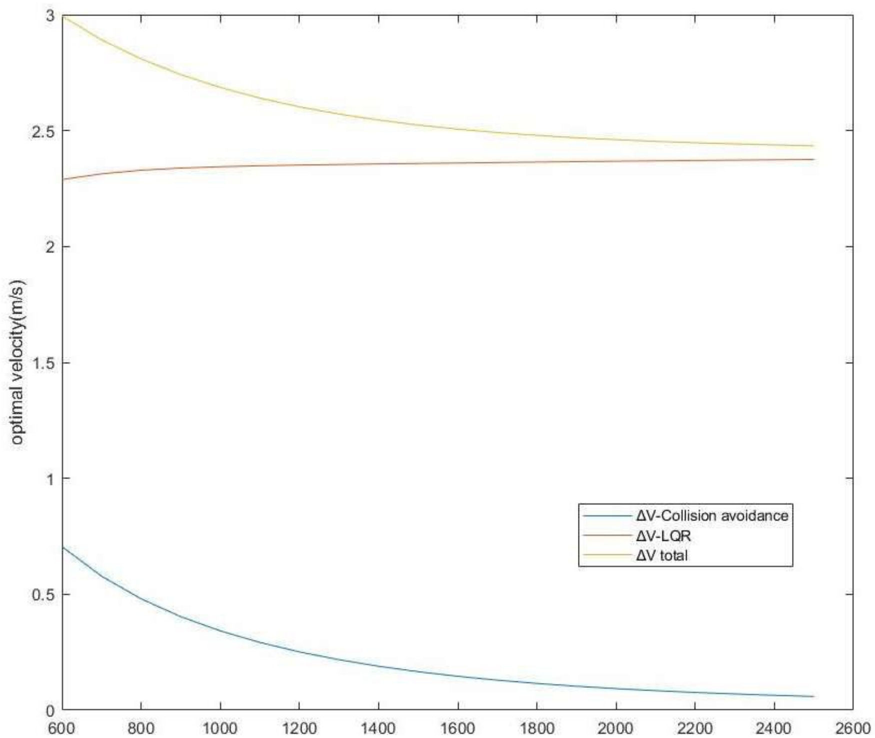
This means once an anti-collision manouever is performed a reacquisition manouever becomes neccessary to get them back to the initial relative orbit. This can be acheived using different techniques, like:

1.A second Impulse manoeuver

2.A continous control manoeuver, and specifically an optimal one as the Linear Quadratic Regulator (LQR)

Re-Acquisition of the Slot / LQR

- usually applies to linear, or linearized systems (we can use HCW)



Manoeuvre anticipation time(s)

$$\dot{X} = Ax + Bu$$

$$J(x,u) = \frac{1}{2}X(t_F)^T S_F X(t_F) + \frac{1}{2}\int_{t0}^{tf} [Xt^T Q X(t) + u^T R u]dt$$

- The total cost for the manoeuver varies almost entirely with the manoeuver anticipation time.
- It can be seen that the cost of the LQR manoeuver is almost same and does not vary much with how early a manoeuver is perfomed
- Example of total Δv - constraint on miss distance is 0.5km

Collision avoidance manouevers for cubesats

- The common CUBESAT thrusters available are usually able to perform the formation flying manoeuvers.

- Solid propellants and liquid propellants usually only require an impulse to perform the manoeuver.

- Low thrust propulsions usually require a continuous velocity manoeuvre to perform the anti-collision manoeuver.

- This continuous velocity manoeuvre also induces a small penalty compared to the impulse manoeuver. Taking the penalty into account it can be expected that the result of control manoeuver will be slightly degraded.

A practical example of collision avoidance manouever

Busek Micro-resistojet thruster	
Nominal thrust	2-10 mN
ISP	150s
Mass	<1.25kg
Mass of the satellite	6kg

- A Busek Micro-Resistojet thruster suitable for cubesats is considered for applying a collision avoidance manoeuver in formation flying satellites.
- The helix relative trajectory of TANDEM-X satellites is considered. The mass is assumed to be an equivalent of the cubesat.
- The time required for the manoeuver is given by $\mathrm{tb} = \frac{M_E * V_E}{T} * \left(1 - e^{-\frac{\Delta V}{V_E}}\right)$

where mi is the initial mass of the satellite and T is the Thrust

RESULTS OF A PRACTICAL MANOEUVER

MANOEUVER CONDITIONS AND RESULTS(TANDEM-X Formation)

ANTICIPATION TIME	3000s
MISS DISTANCE CONSTRAINT	0.4 km
ΔV IMPULSE MANOEUVER	.03017311 m/s
VELOCITY PENALTY	2.5473396775e-7 m/s
ΔV CONTINOUS MANOEUVER	.03017357 m/s
MISS DISTANCE IMPULSE MANOUEVER	0.400015 km
MISS DISTANCE CONTINOUS MANOEUVER	0.385696 km

- An approximation for velocity penalty for transformation from impulsive to continuous is given by
- $Velocity\ penalty = \frac{1}{24} \times (n \times tb)^2 \times \Delta V_{Impulse}$
- Even with the added penalty we can see that changing the manoeuvre does not give us a perfect Manoeuvre. The distance constraint is violated ever so slightly

Final Remarks

- Collision avoidance will become a must not only for large platforms yet also for small satellites, especially advanced cubesat and those flying in formation.

- Unfortunately small platforms have limited capabilities in terms of maneuvering. To take into account the need for collision avoidance, and optimize their design is an important aspect

- This presentation depicted some preliminary work on the subject, starting from impulsive maneuver approximation. Thrust level was selected to be suitable for cubesats. Penalty for continuous vs. impulsive maneuver was considered

- Troubles increase when platforms are flying in formation, as the orbital geometry should be re-acquired after the maneuver to satisfy mission requirements. LQR has been evaluated for this second phase, while first collision avoidance maneuver is the leading contribution in Δv