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Attitude Control Algorithms in a Swarm of CubeSats: Kriging Interpolation and Coordinated Data Exchange

5th IAA Conference on University Satellite Missions and CubeSat Workshop

participants: **Anton Ivanov**
Ahmed Mahfouz
Dmitry Pritykin

presenter: **Anton Afanasev**
PhD student, Space Center
Skoltech, Moscow

What's the point?

1

Current approaches to ADCS: star sensors, sun sensors, gyros, magnetometers and their combinations

Best orientation accuracy: 0.01°

Cost/dimension/mass budget: high

2

Question: What if we use only magnetometers?

Answer: Accuracy is on average 10-15°. Unacceptable!

3

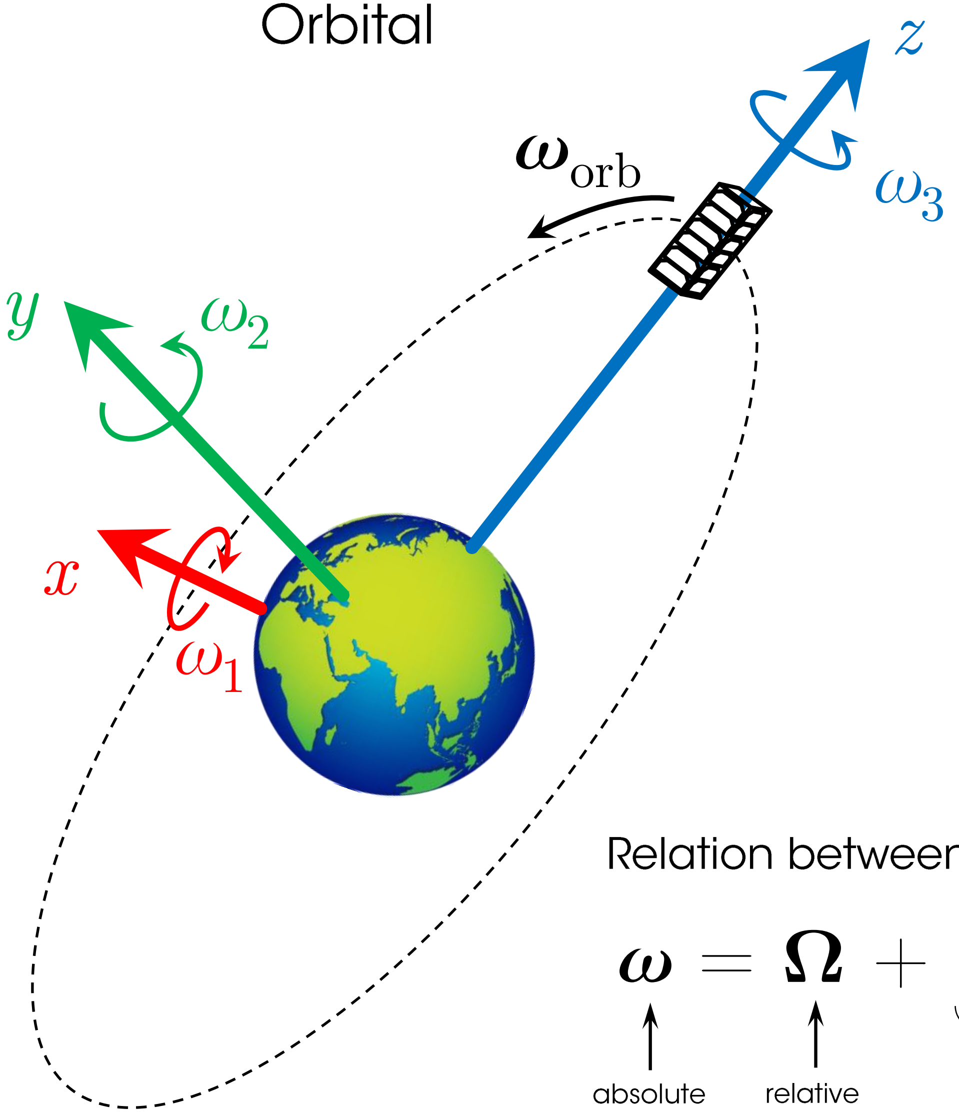
Proposal of this article: To use magnetometer readings in Swarm of CubeSats and optimize their measurements and orientation via data exchange between them and the interpolation of magnetic field

Expected accuracy: Less than 10°

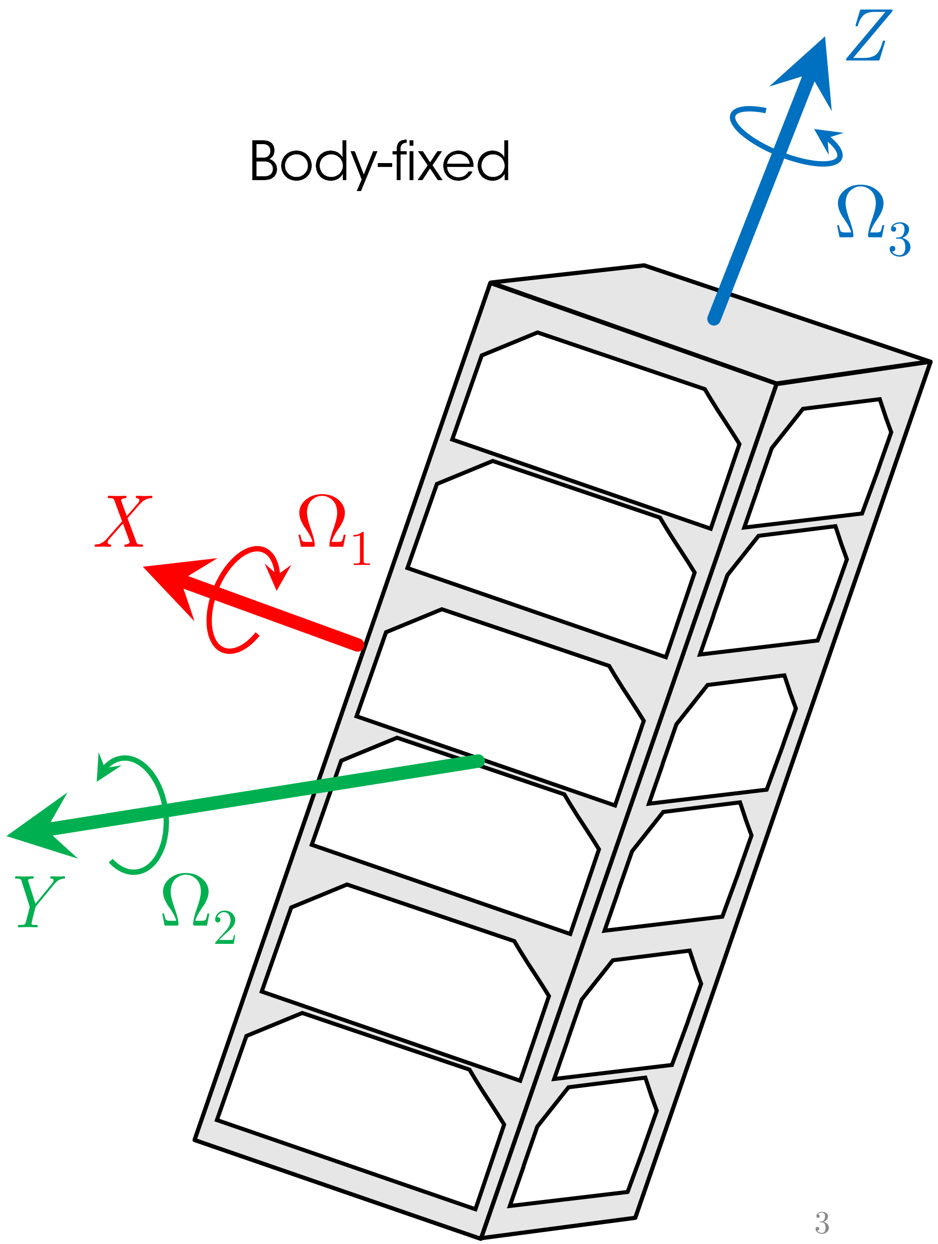
Benefits: Low cost, small size, light, low energy consumption

Reference frames

Orbital



Body-fixed



Relation between angular velocities:

$$\underset{\text{absolute}}{\boldsymbol{\omega}} = \underset{\text{relative}}{\boldsymbol{\Omega}} + \underbrace{\tilde{Q} \circ \boldsymbol{\omega}_{\text{orb}} \circ Q}_{\text{frame motion}}$$

Motion equations

Rigid body dynamics (Euler eq.)

$$\mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega},\, \mathbf{J}\boldsymbol{\omega}] = \mathbf{T}$$

Quaternion kinematics (Poisson eq.)

$$\dot{\mathbf{q}} = \frac{1}{2}(q_0\boldsymbol{\Omega} + [\mathbf{q},\, \boldsymbol{\Omega}])$$

$$\dot{q}_0 = -\frac{1}{2}\mathbf{q}^T\boldsymbol{\Omega}$$

Notations:

- $\mathbf{J}$ – inertia tensor
- $\boldsymbol{\omega}$ – absolute angular velocity
- $\boldsymbol{\Omega}$ – relative angular velocity
- q_0 – scalar component of quaternion Q
- $\mathbf{q}$ – vector component of quaternion Q
- $\mathbf{T}$ – external torque

orientation parameters ($\boldsymbol{\omega}$, $\boldsymbol{\Omega}$, q_0, $\mathbf{q}$)

We can manipulate the motion of satellite via external torque $\mathbf{T}$

Motion equations

Rigid body dynamics (Euler eq.)

$$\mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega},\, \mathbf{J}\boldsymbol{\omega}] = \mathbf{T} \quad \longrightarrow$$

Quaternion kinematics (Poisson eq.)

$$\dot{\mathbf{q}} = \frac{1}{2}\left(q_0\boldsymbol{\Omega} + [\mathbf{q},\, \boldsymbol{\Omega}]\right)$$

$$\dot{q}_0 = -\frac{1}{2}\mathbf{q}^T\boldsymbol{\Omega}$$

External torque

$$\mathbf{T} = \mathbf{T}_{\text{control}} + \mathbf{T}_{\text{grav}} + \mathbf{T}_{\text{dist}}$$

- $$\mathbf{T}_{\text{control}} = [\mathbf{m},\, \mathbf{B}]$$

  $\mathbf{m}$ — magnetic moment; $\mathbf{B}$ — external magnetic field

- $$\mathbf{T}_{\text{grav}} = 3\omega_{\text{orb}}^2\, [\mathbf{e}_Z,\, \mathbf{J}\mathbf{e}_Z]$$

  $$\mathbf{e}_Z = \tilde{Q} \circ (0,0,1) \circ Q$$

  $(0,0,1)$ — unit radius-vector Oz in the orbital frame

- $$\mathbf{T}_{\text{dist}} = \eta_{\text{torque}}(0,\, \sigma_{\text{torque}}^2)$$

Control magnetic moment

Lyapunov-based controller

$$\mathbf{m} = k_\omega[\Delta\boldsymbol{\Omega},\, \mathbf{B}] + k_a[\Delta\mathbf{S},\, \mathbf{B}]$$

$\Delta\boldsymbol{\Omega}$ – difference between current angular velocity and desired one

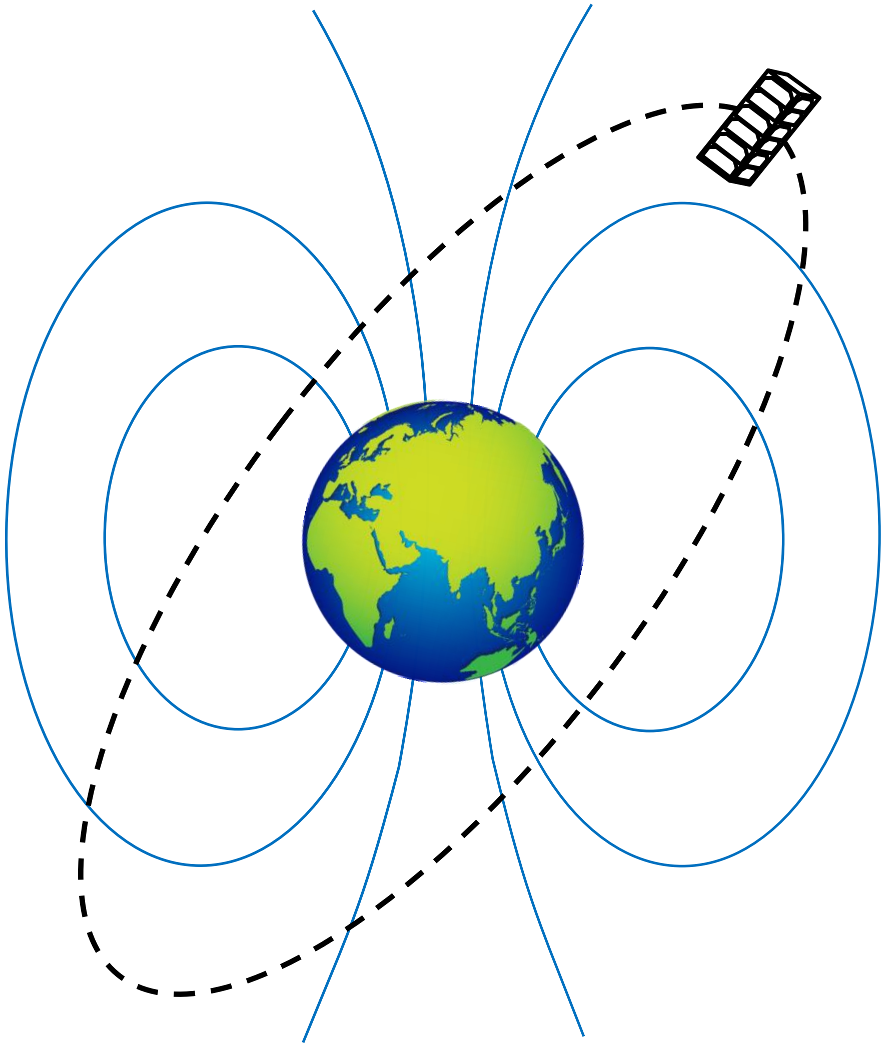
$\Delta\mathbf{S}$ – difference between current orientation and desired one

$\mathbf{B}$ – external magnetic field

$k_\omega,\ k_a$ – coefficients of proportionality

D.S.Ivanov et al. Advanced numerical study of the three-axis magnetic attitude control and determination with uncertainties.

Magnetic field model

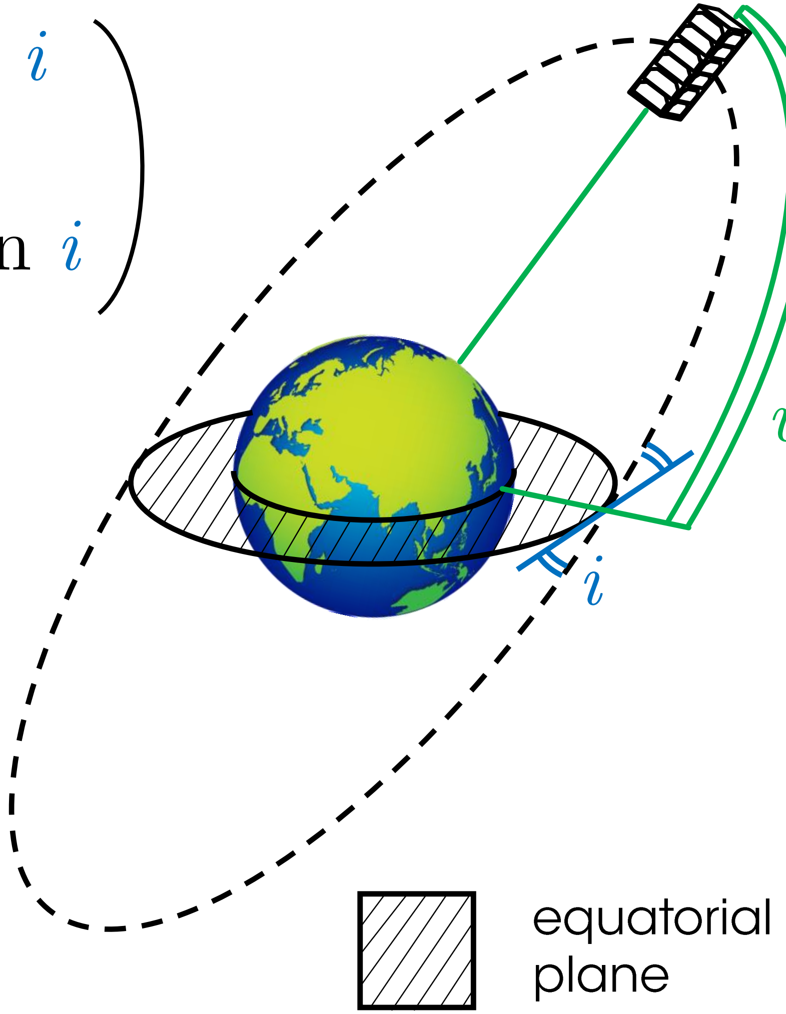


$$\mathbf{B}_{\text{orb}} = B_0 \begin{pmatrix} \cos\, u \sin\, i \\ \cos\, i \\ -2\sin\, u \sin\, i \end{pmatrix}$$

u – argument of latitude in the orbit

i – inclination of the orbit

B_0 – magnitude of the magnetic field on the orbit



Measurements model

Magnetometer noise

$$\mathbf{B}_{\text{meas}} = \tilde{Q} \circ \mathbf{B}_{\text{orb}} \circ Q + \eta_{\text{meas}}(\mathbf{B}_{\text{bias}},\ \sigma^2_{\text{meas}})$$

$$\dot{\mathbf{B}}_{\text{bias}} = \eta_{\text{bias}}(0,\ \sigma^2_{\text{bias}})$$

hard mode

Actual magnetic field

$$\mathbf{B}_{\text{env}} = \tilde{Q} \circ \mathbf{B}_{\text{orb}} \circ Q + \eta_{\text{artificial}}(0,\ \sigma^2_{\text{artificial}})$$

current approach

Actual field should be taken from databases, such as IGRF–12
https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html

CubeSat dynamics

Angular velocity of the CubeSat in the **orbital reference frame** →

Angular velocity of the CubeSat in its **own reference frame** →

Euler angles of the CubeSat →

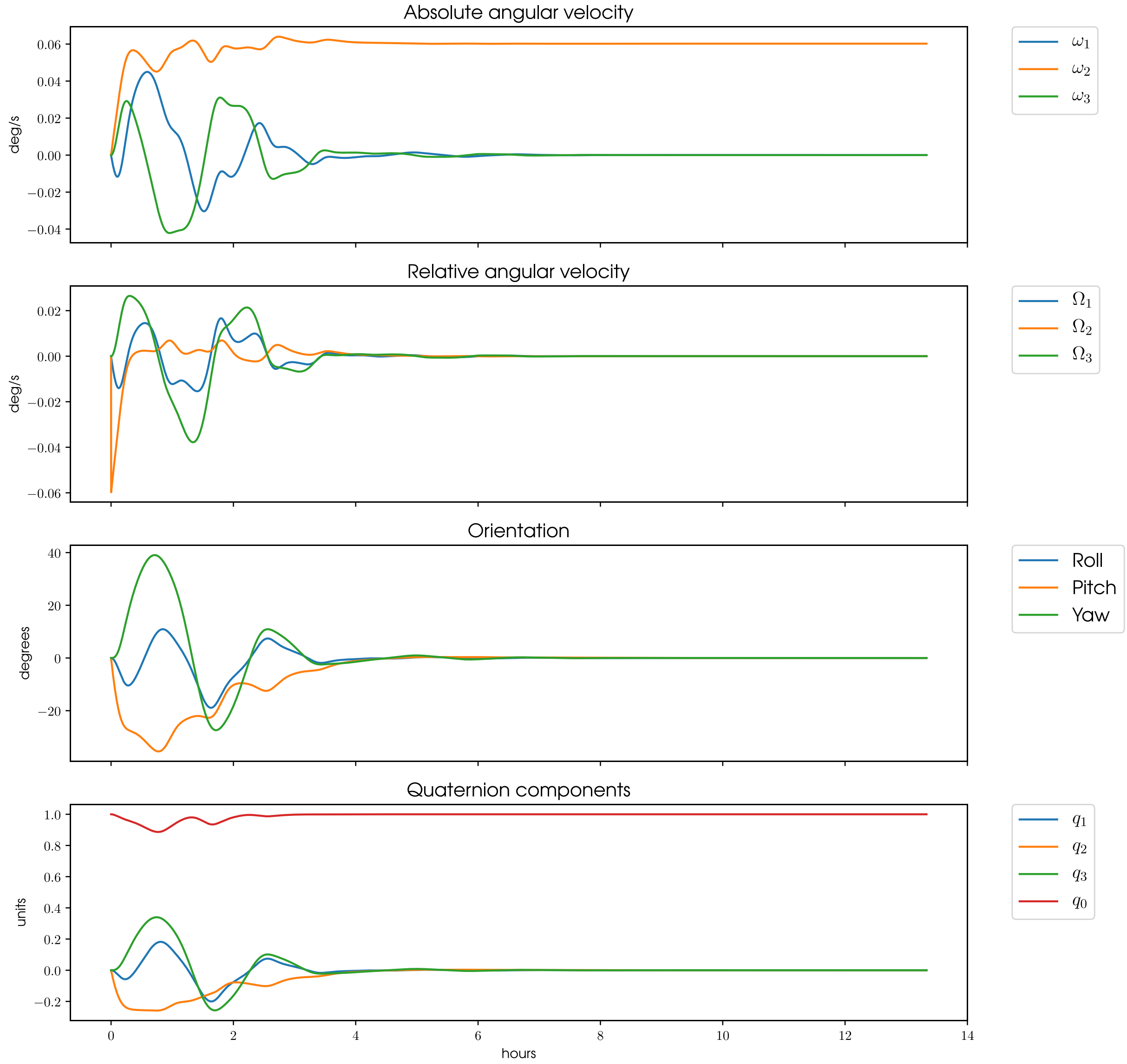
Quaternions of the CubeSat →

DYNAMICS

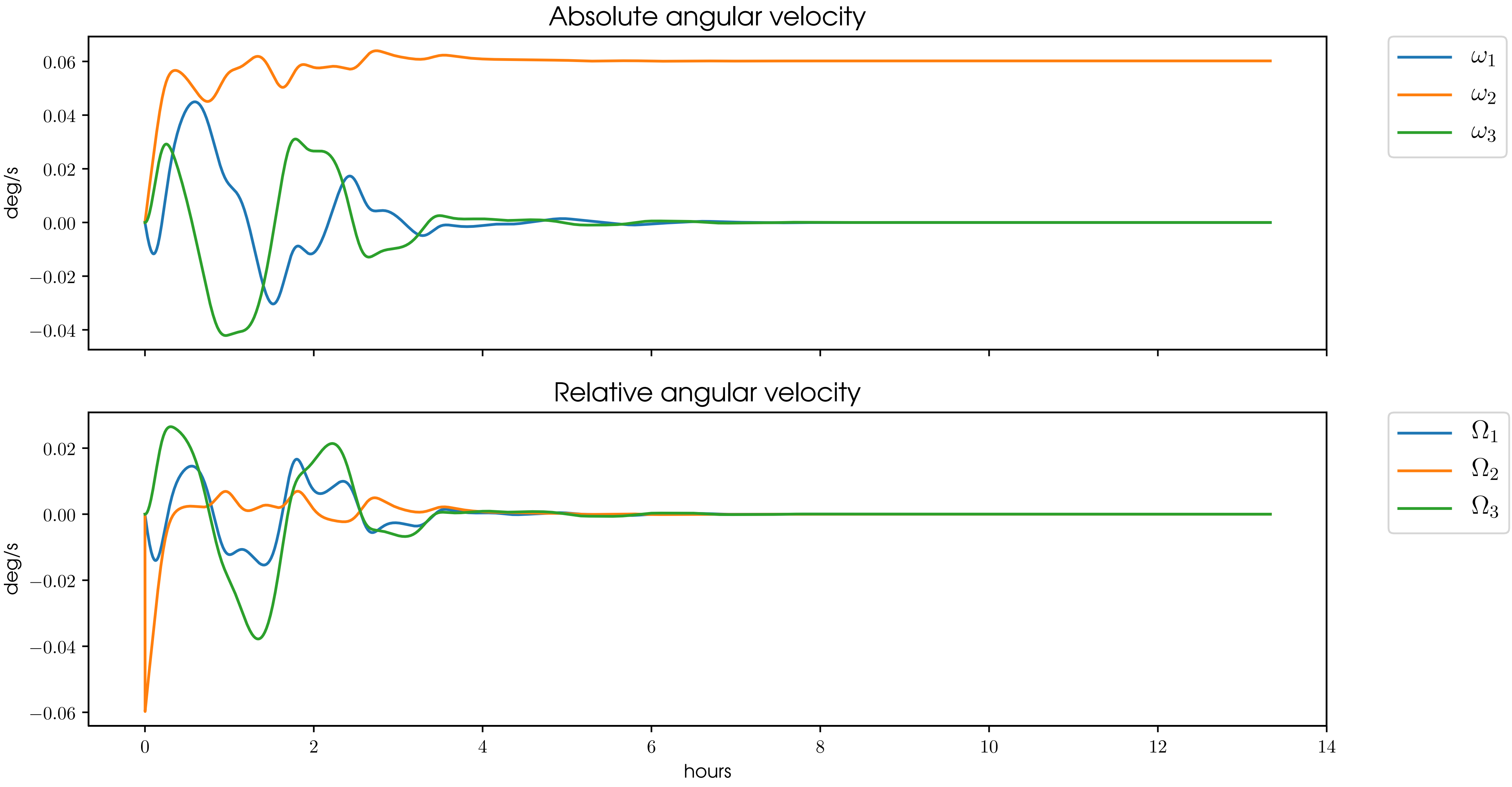
$q^{\text{init}} = (1.0, 0.0, 0.0, 0.0)$ $\quad \omega^{\text{init}} = (0.0, 0.0, 0.0)$ [rad/s]

$J = \text{diag}(0.011, 0.014, 0.009)$ [kg·m²] $\quad (k'_\omega, k_a) = (60, 12)$ [N·m/T²] $\quad \mu_{\max} = 0.1$ [A·m²]

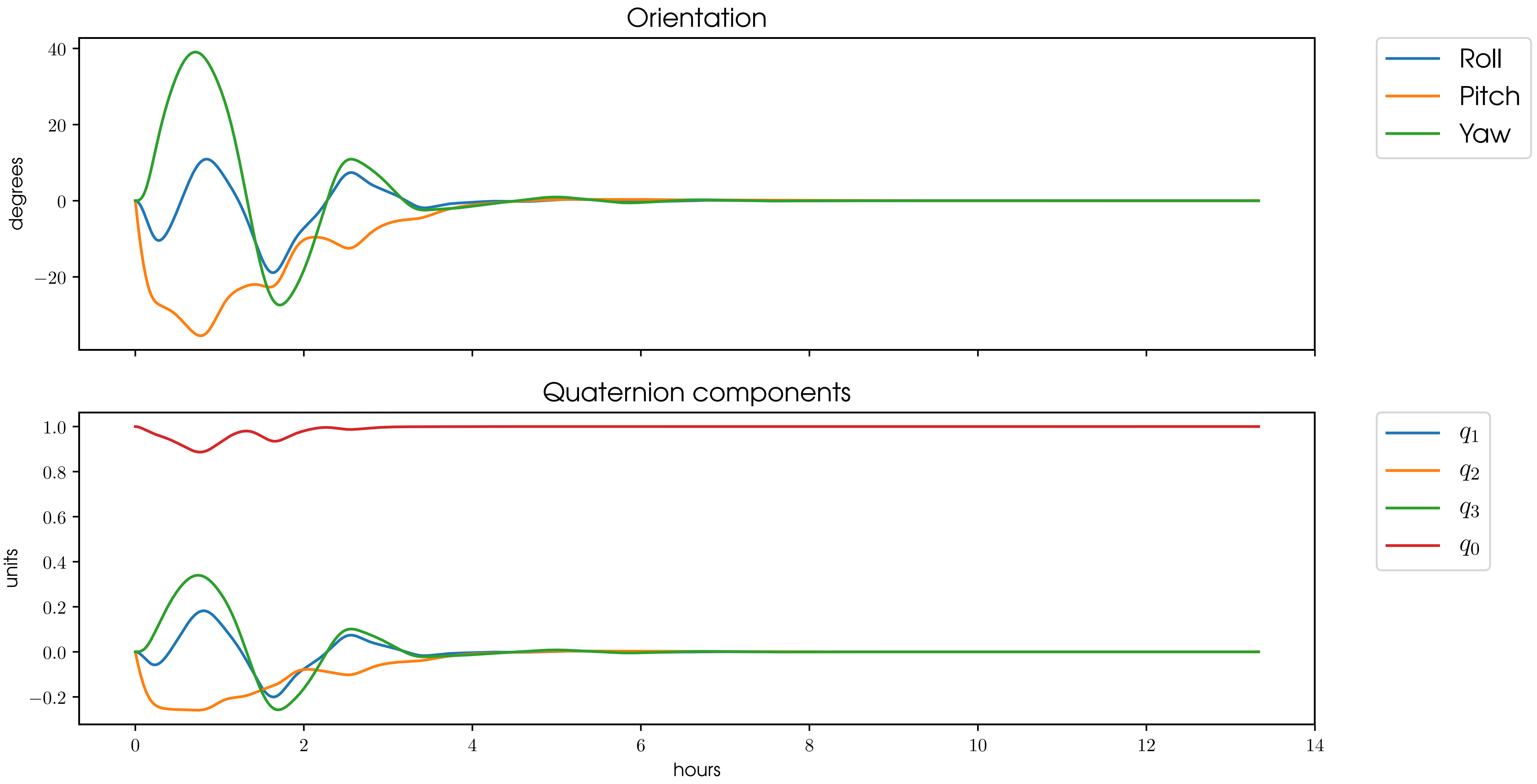
$h_{\text{orb}} = 750$ [km] $\quad i = 60.0^\circ$ $\quad (T_1, T_5) = (1.0, 5.0)$ [s]



CubeSat angular velocities (no disturbances)



CubeSat orientation (no disturbances)



Extended Kalman filter

Predicting state estimate

$$\tilde{X}_i = f(\hat{X}_{i-1})$$

Correcting state estimate

$$\hat{X}_i = \mathbf{K}_i z_i + (\mathbf{I} - \mathbf{K}_i h)\tilde{X}_i$$

Coefficients K_i are chosen in such way that

$$\mathbf{E}\left[||\hat{X}_i - X_i||^2\right] \rightarrow \min$$

all technical story is buried here

Notations:

- X_i – true state vector
- $\tilde{X}_i$ – predicted state estimate
- $\hat{X}_i$ – corrected state estimate
- z_i – measurement
- $\mathbf{K}_i$ – gain matrix
- $\mathbf{I}$ – identity matrix
- $\mathbf{E}$ – expected value (mean)

Physical imperfections

It takes some time to execute the control action $X_i = f(X_{i-1}) \leftarrow T_{\mathrm{ctrl}} = 5$ sec

and read with magnetometer $z_i = h(X_i) \leftarrow T_{\mathrm{meas}} = 1$ sec

Magnetorquers can generate limited amount of magnetic moment $\leftarrow \mu_{\max} = 0.1$ A•m^2

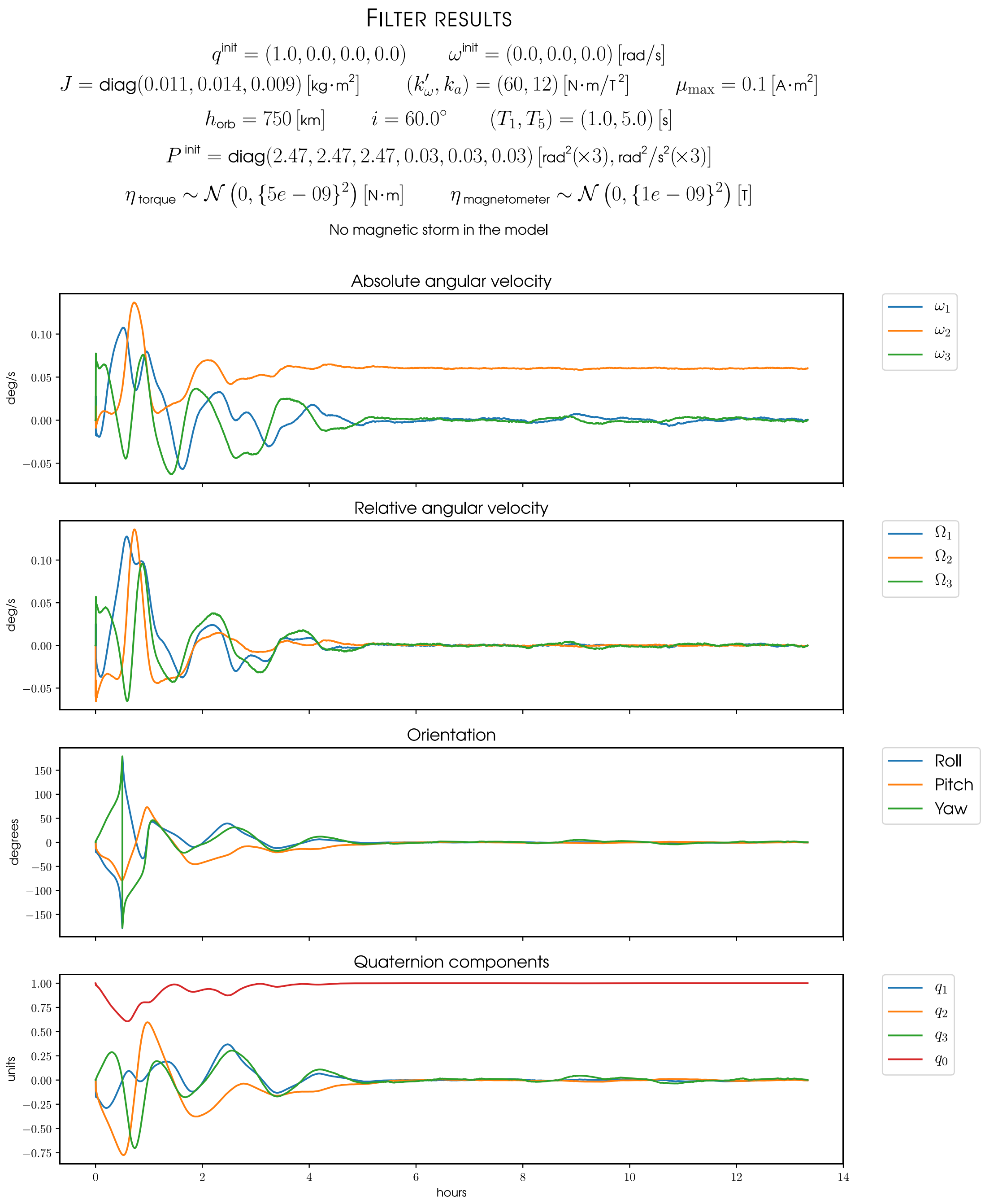
Simulation results

$$\sigma_{\text{torque}} = 5 \text{ nN·m}$$

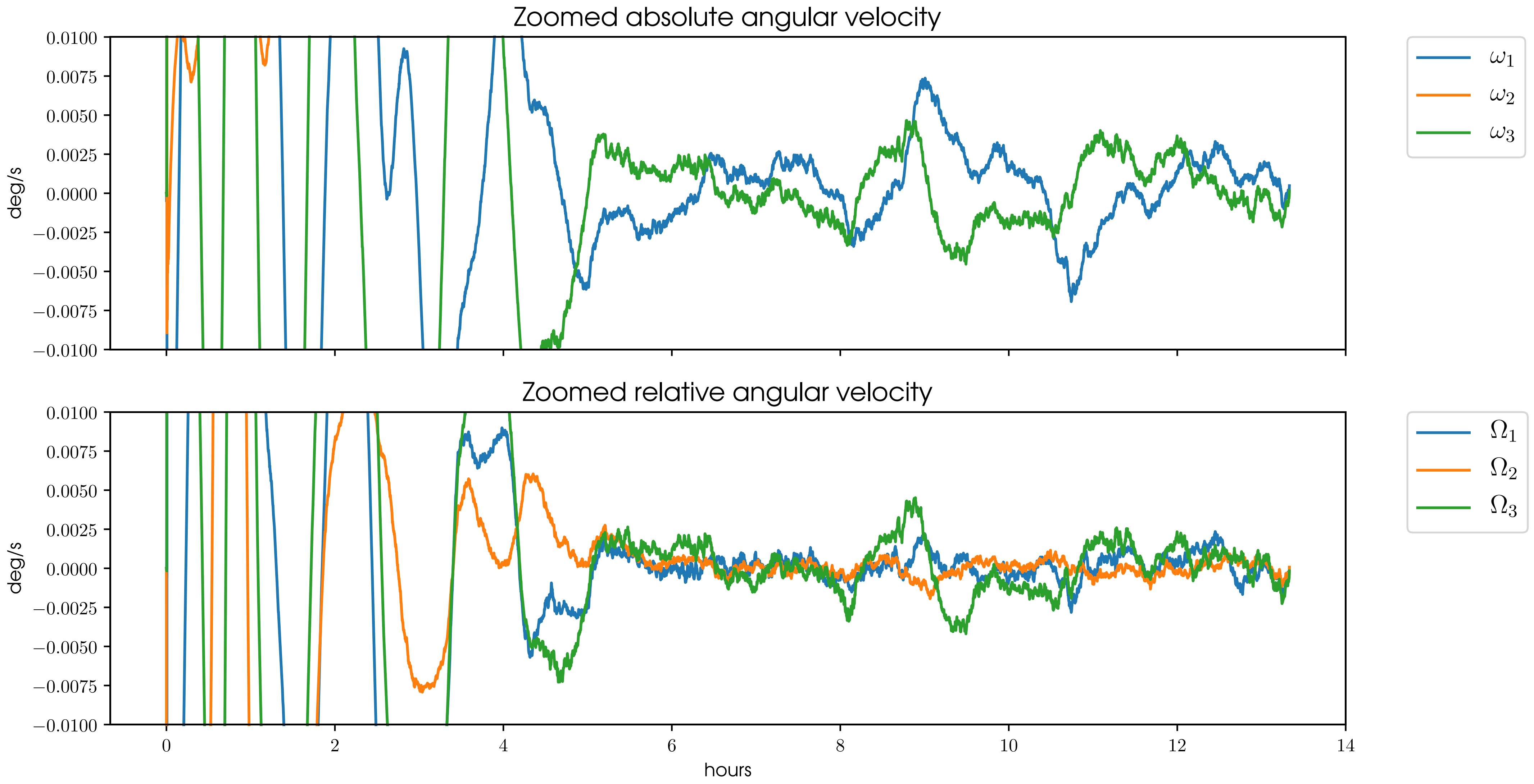
$$\sigma_{\text{meas}} = 1 \text{ nT}$$

$$\mathbf{B}_{\text{bias}} = 0$$

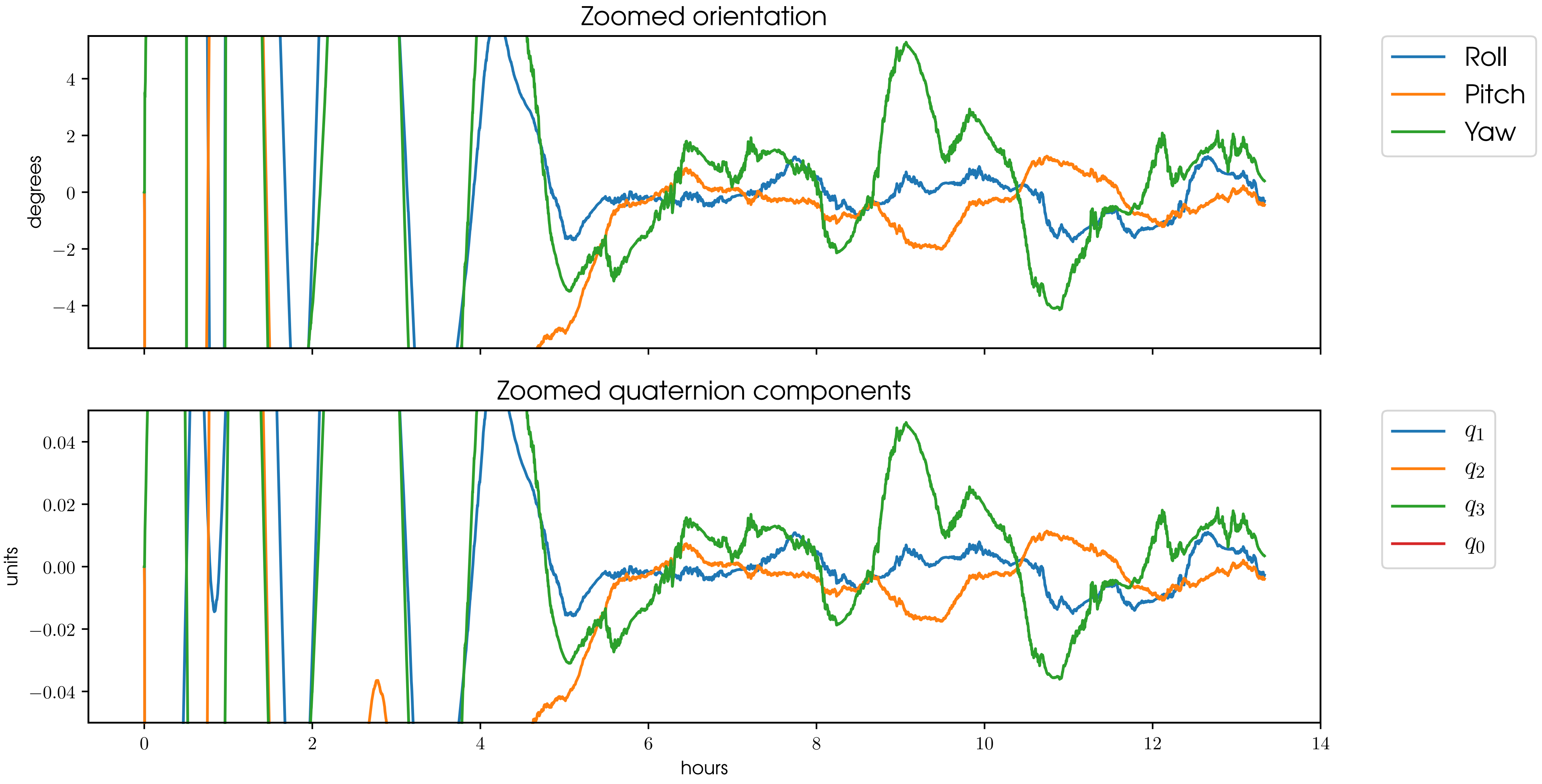
$$\sigma_{\text{artificial}} = 1 \text{ nT}$$



CubeSat angular velocities



CubeSat orientation



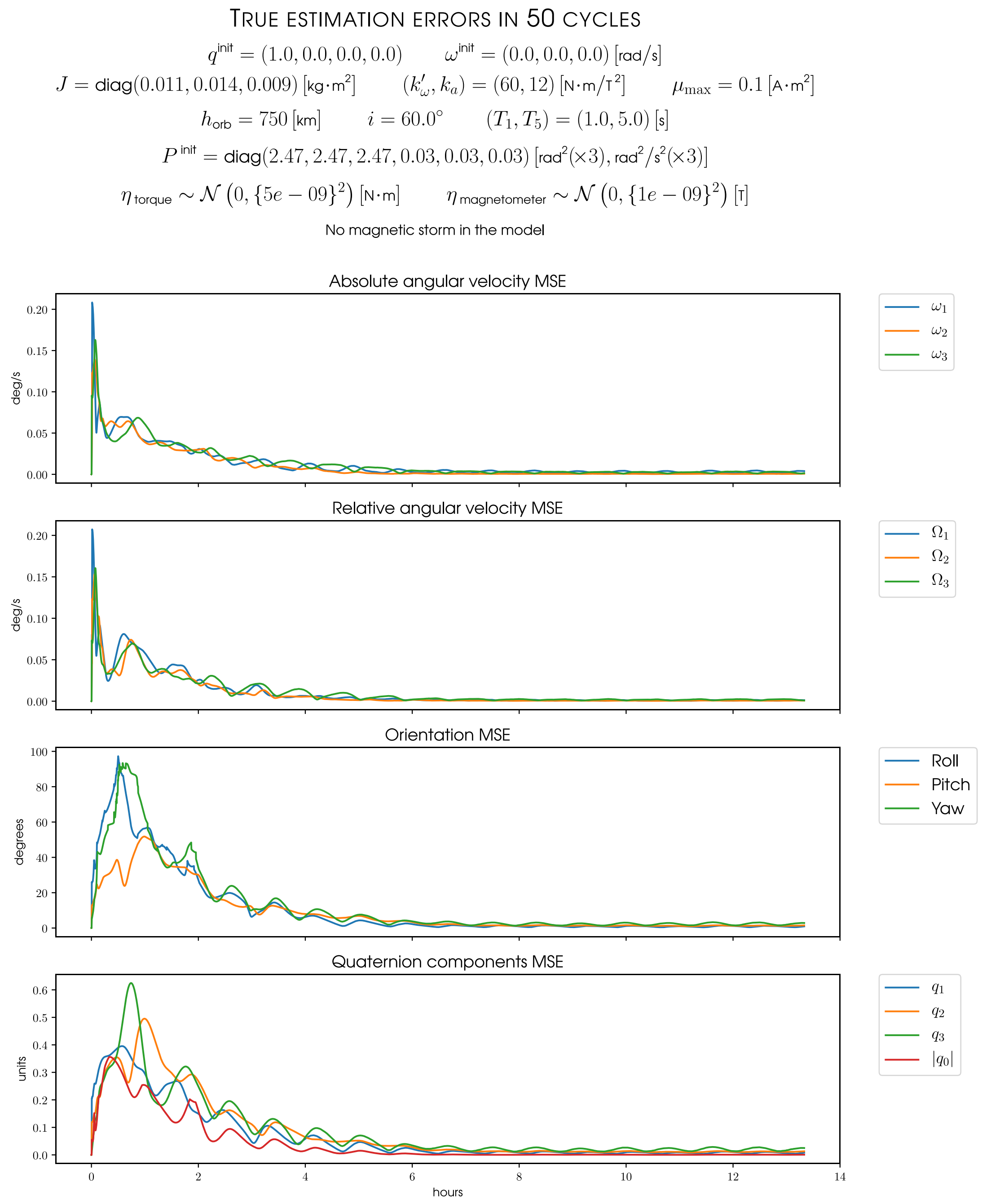
True estimation errors

$$\sigma_{\text{torque}} = 5 \text{ nN·m}$$

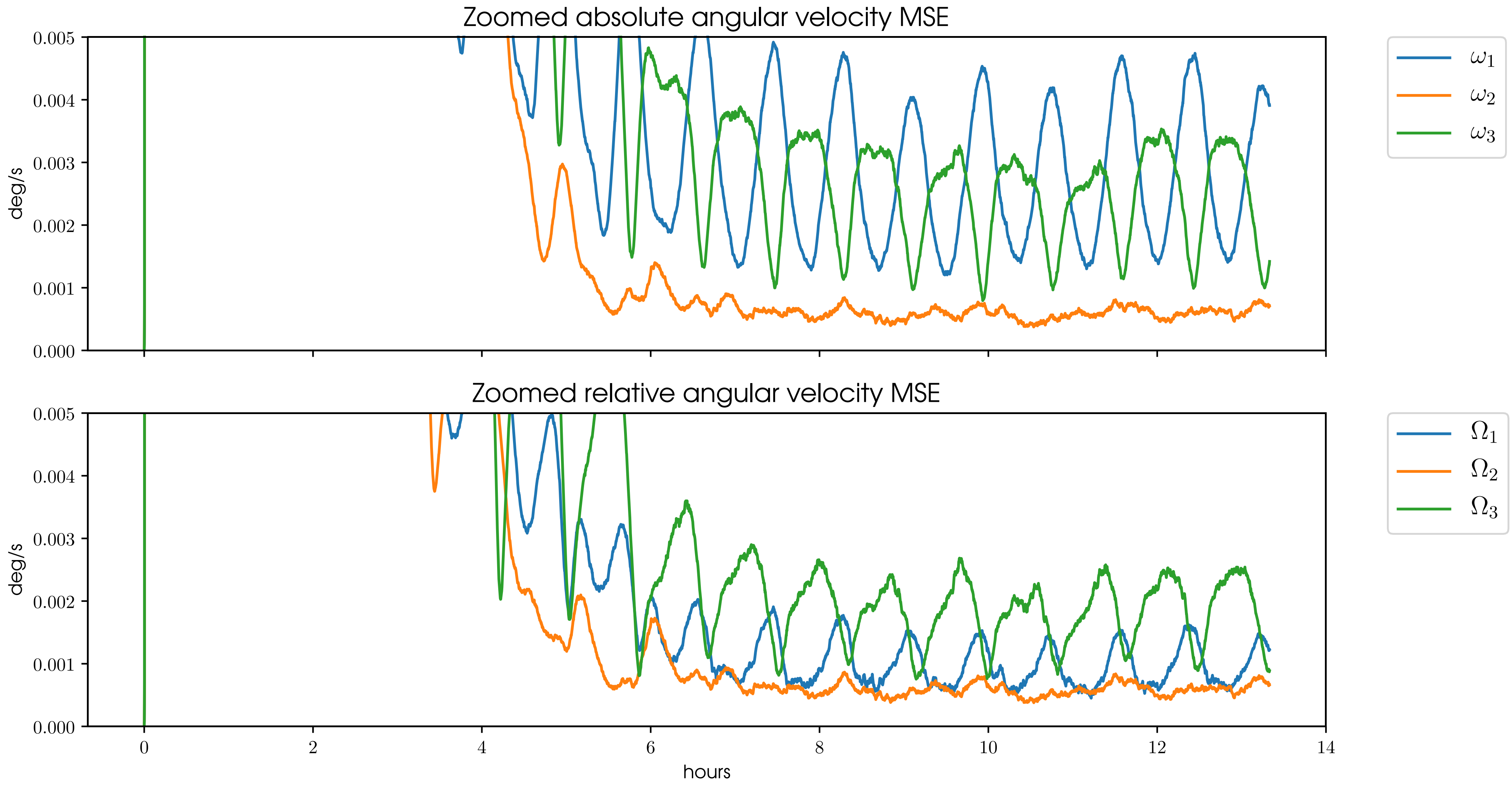
$$\sigma_{\text{meas}} = 1 \text{ nT}$$

$$\mathbf{B}_{\text{bias}} = 0$$

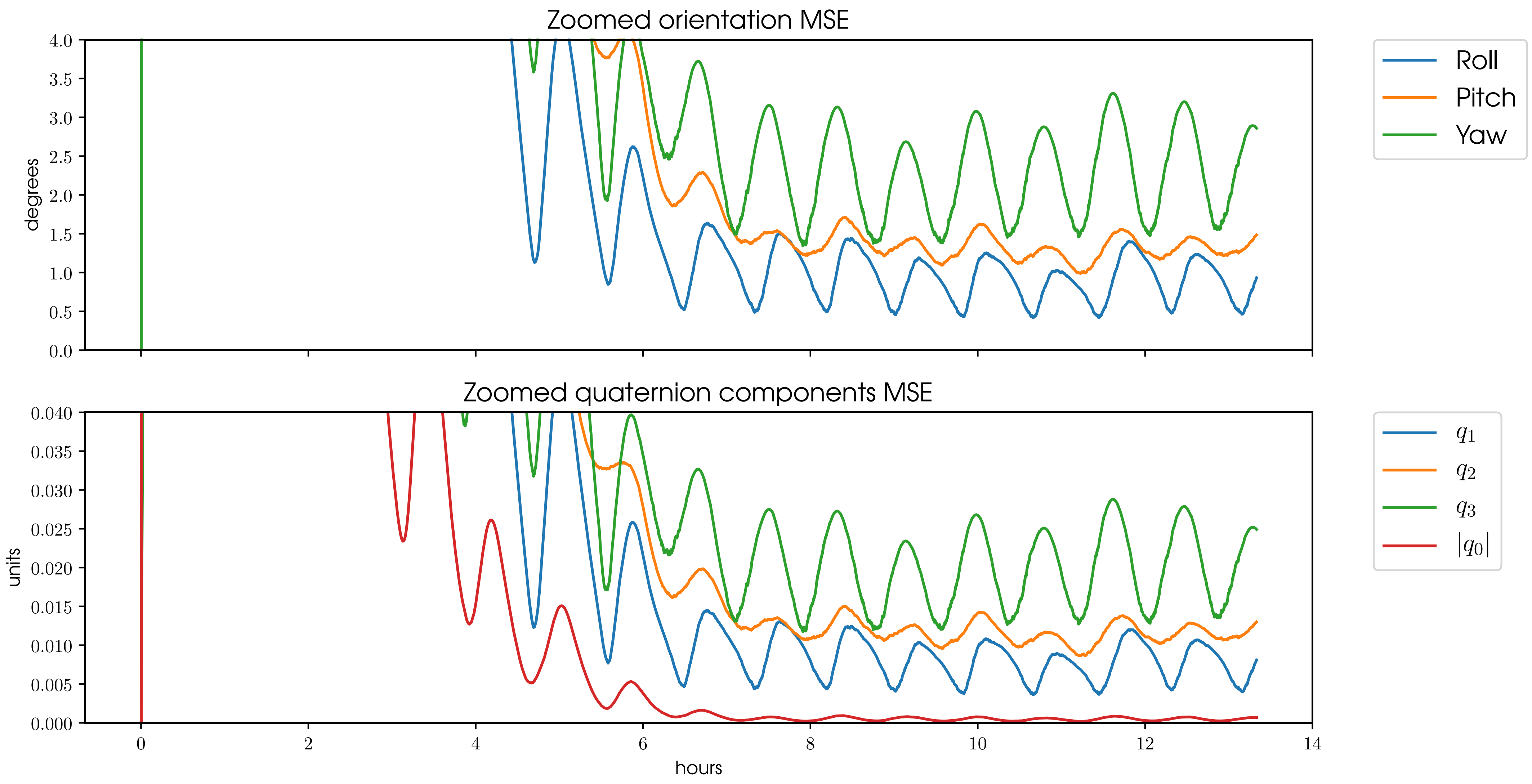
$$\sigma_{\text{artificial}} = 1 \text{ nT}$$



CubeSat angular velocities Mean-Squared Errors



CubeSat orientation Mean-Squared Errors



Swarm structure

Right tetrahedron

Sat1: $(3r, \quad 0, \quad 0)$

Sat2: $(-r, -2\sqrt{2}r, \quad 0)$

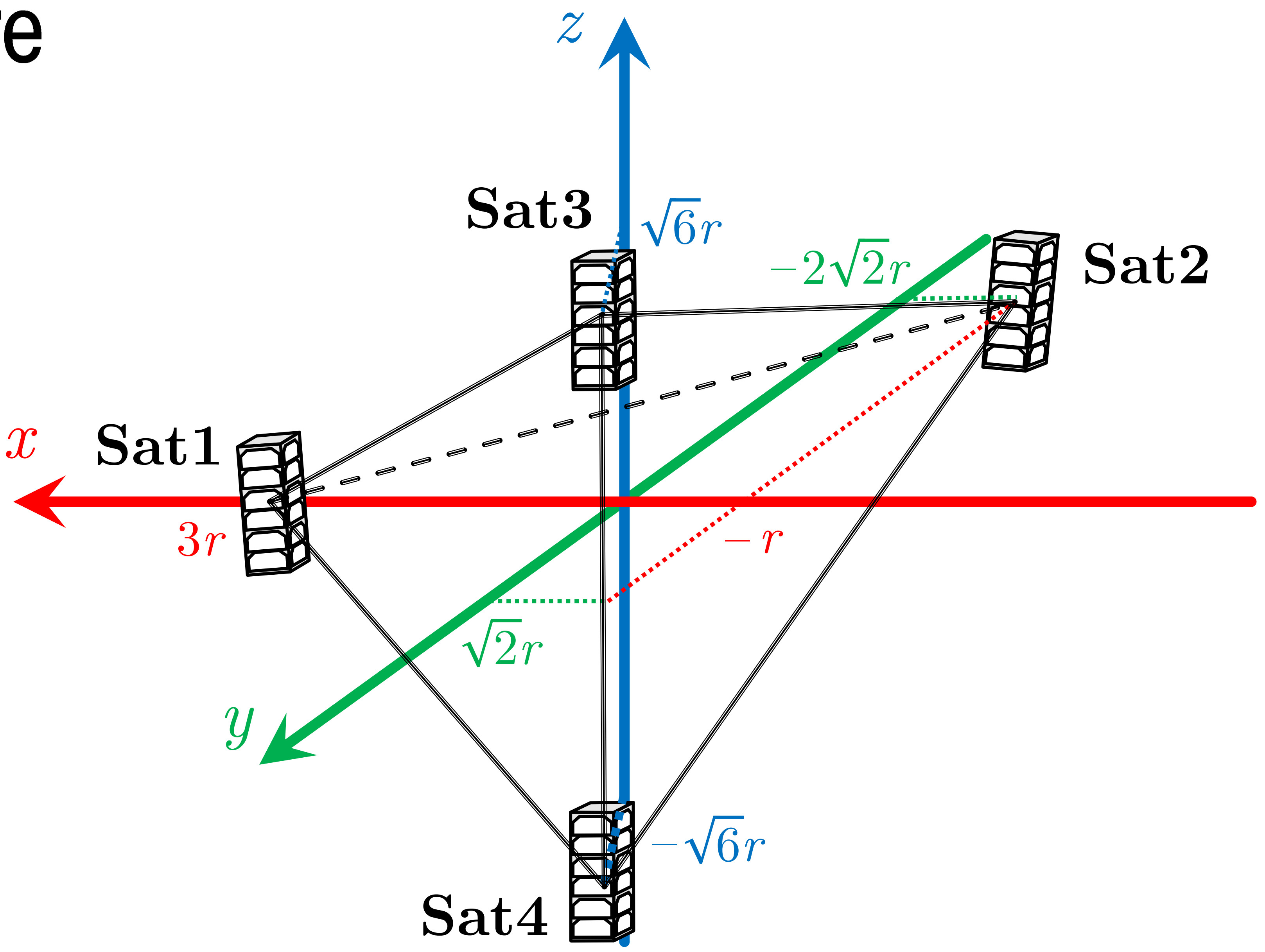
Sat3: $(-r, \quad \sqrt{2}r, \quad \sqrt{6}r)$

Sat4: $(-r, \quad \sqrt{2}r, -\sqrt{6}r)$

Size of tetrahedron is $2\sqrt{6}r$

$\downarrow$

50 km



Moves as a rigid body!

Data exchange

Center of mass behaves as the virtual CubeSat

↓

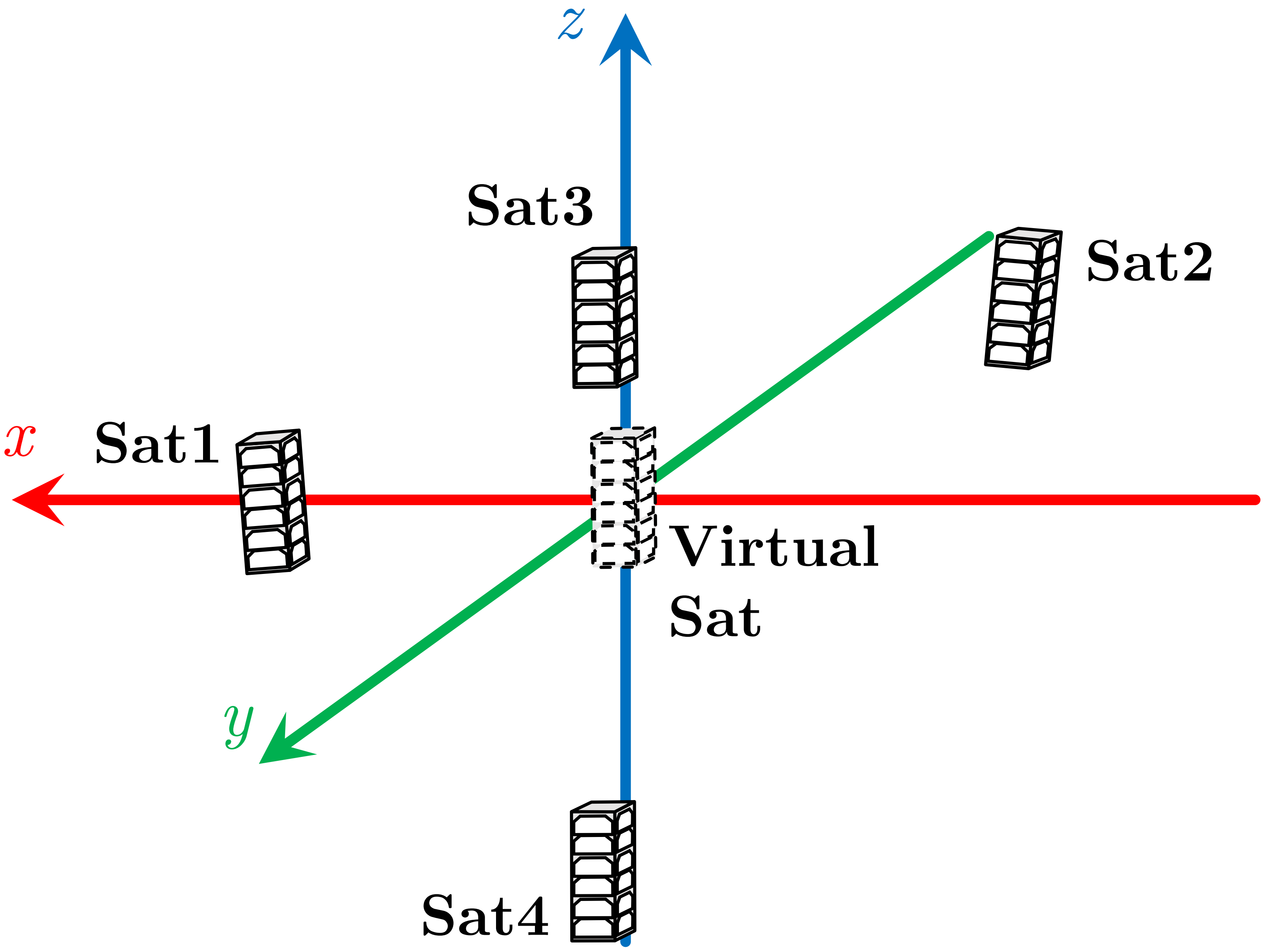
Measurements of magnetic field should be interpolated in common ref. frame

↓

Measurements are interpolated in center of mass ref. frame

↓

Enhanced measurements are sent back to CubeSats



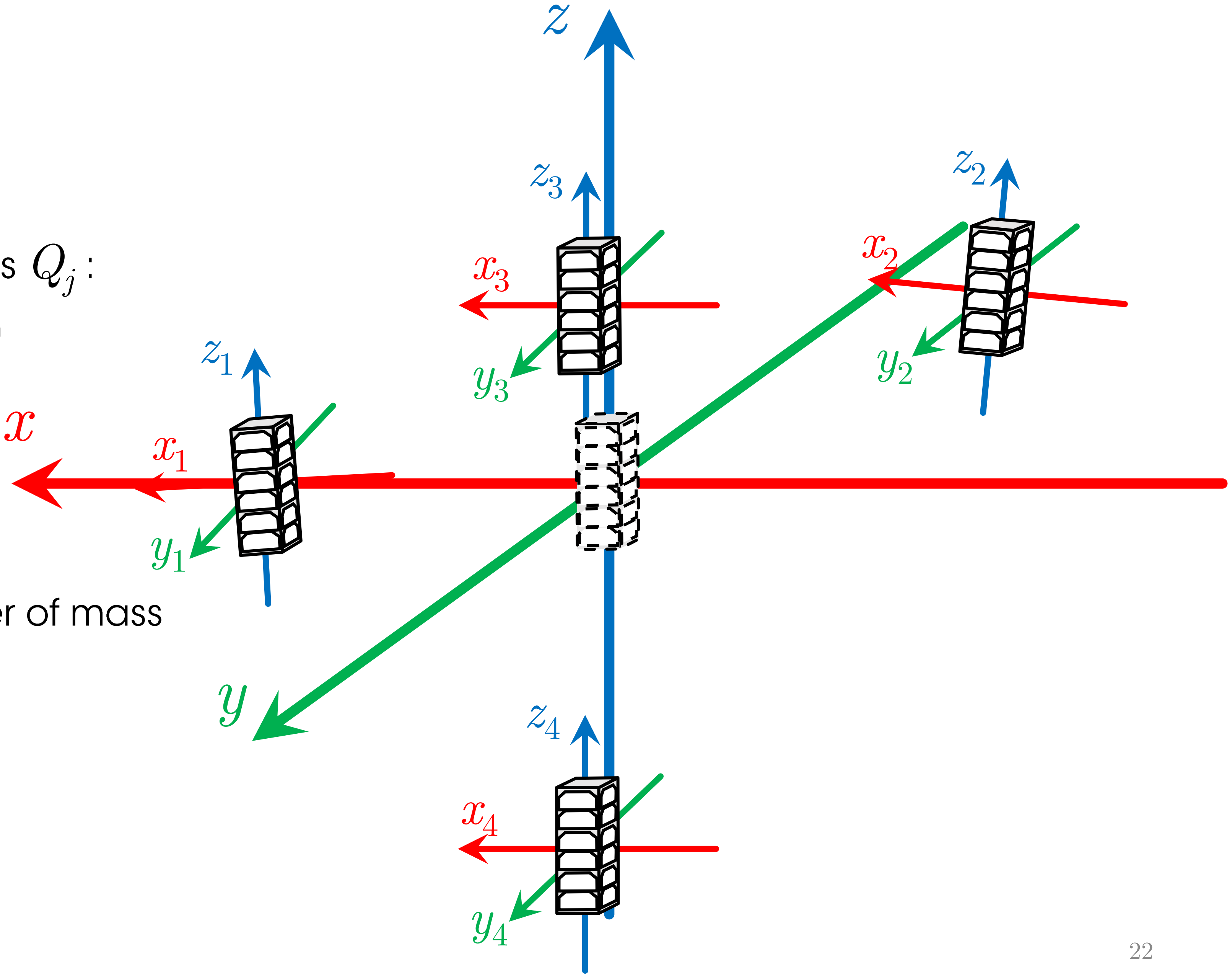
Data exchange

Suppose we know quaternions Q_j :

Orbital j[th] ↔ Body-fixed j[th]

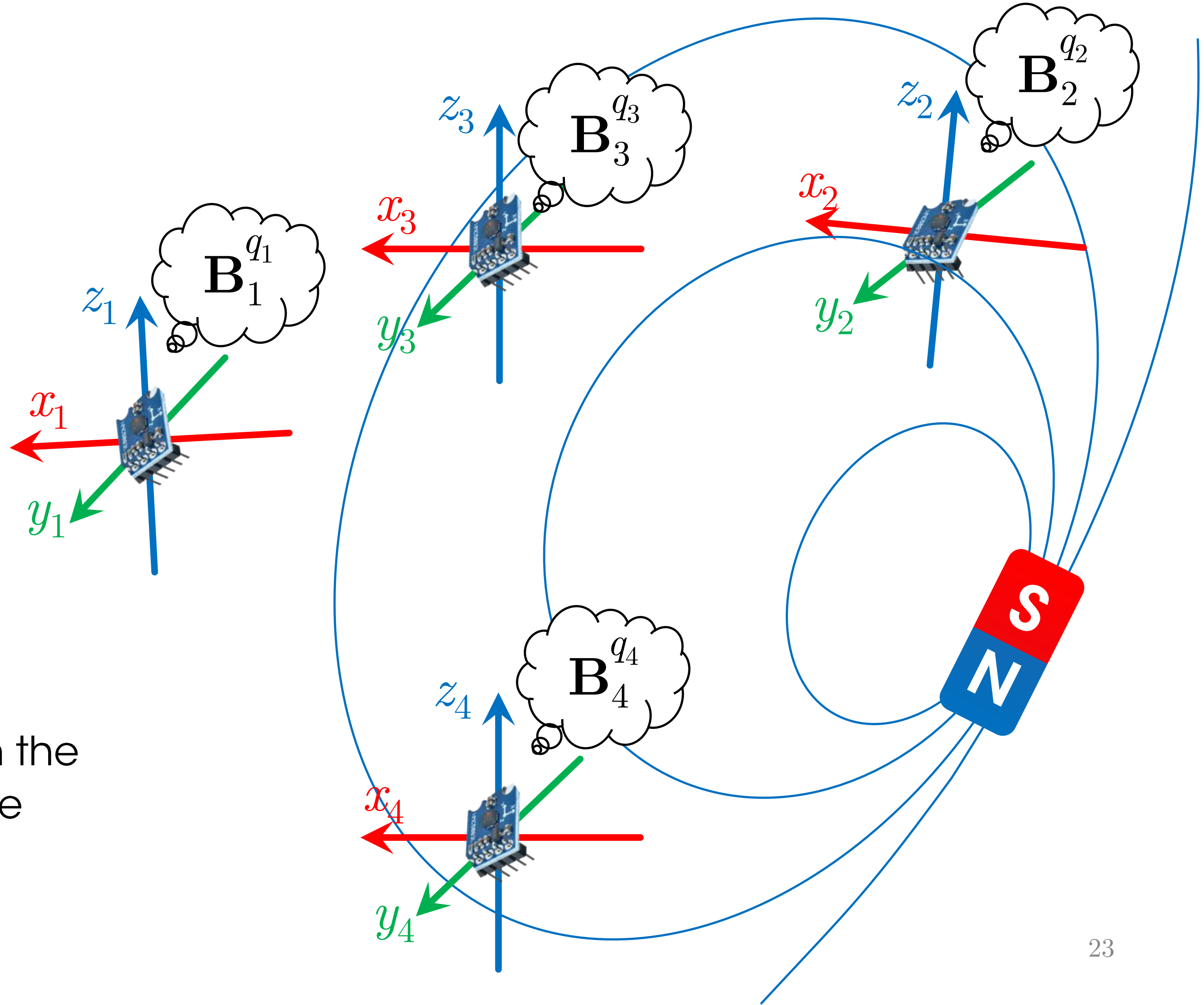
And DCM $\mathbf{A}^{u}_{j\leftrightarrow 0}$:

Orbital j[th] ↔ Orbital center of mass



Data exchange

Measure magnetic field $\mathbf{B}_j^{q_j}$ with magnetometers (body-fixed frame)



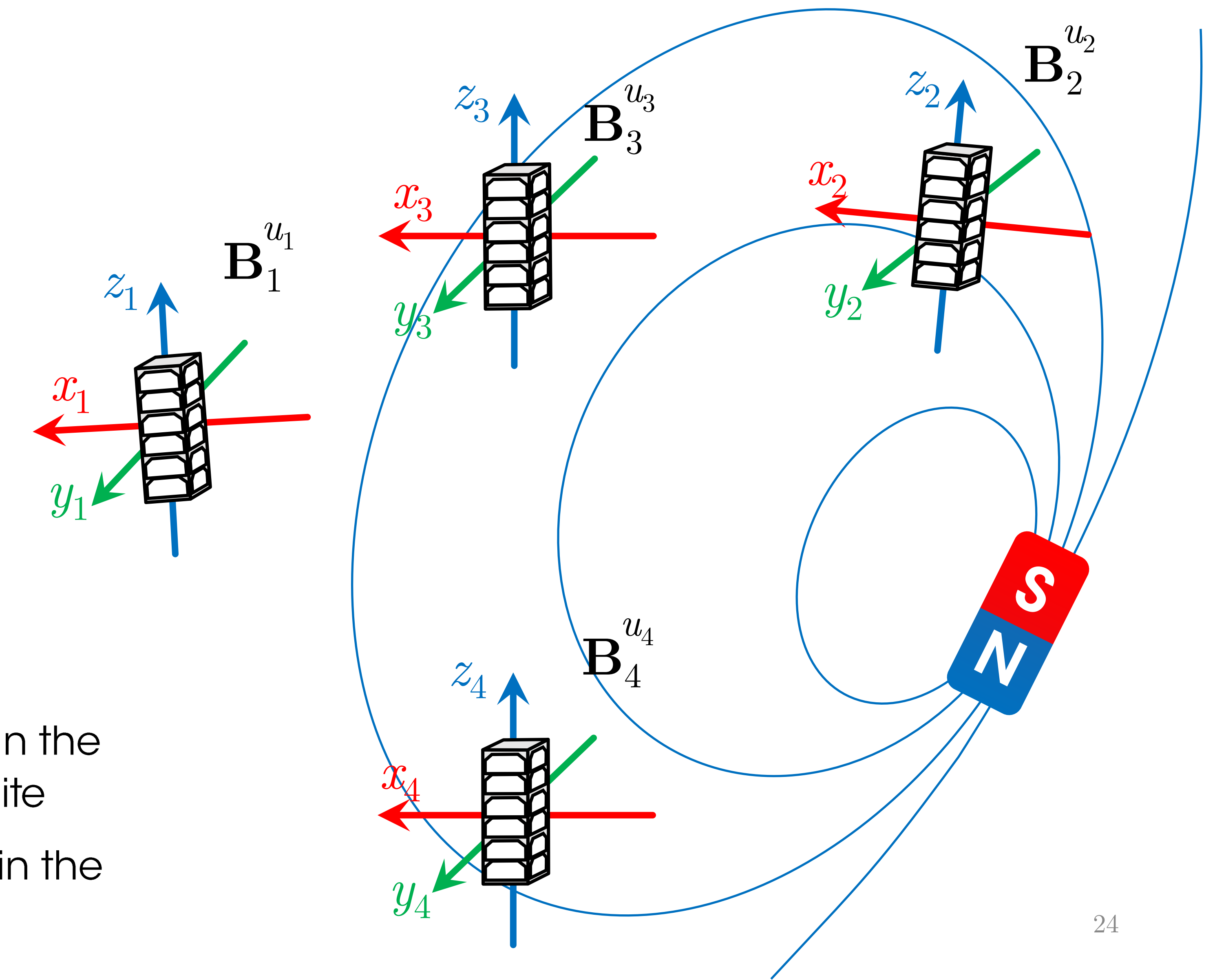
where:

$\mathbf{B}_j^{q_j}$ – magnetic field of j^th satellite in the **body-fixed** frame of j^th satellite

Data exchange

With known quaternion Q_j transform measurement to the orbital frame of j[th] satellite:

$$\mathbf{B}_j^{u_j} = Q_j \circ \mathbf{B}_j^{q_j} \circ \tilde{Q}_j$$



where:

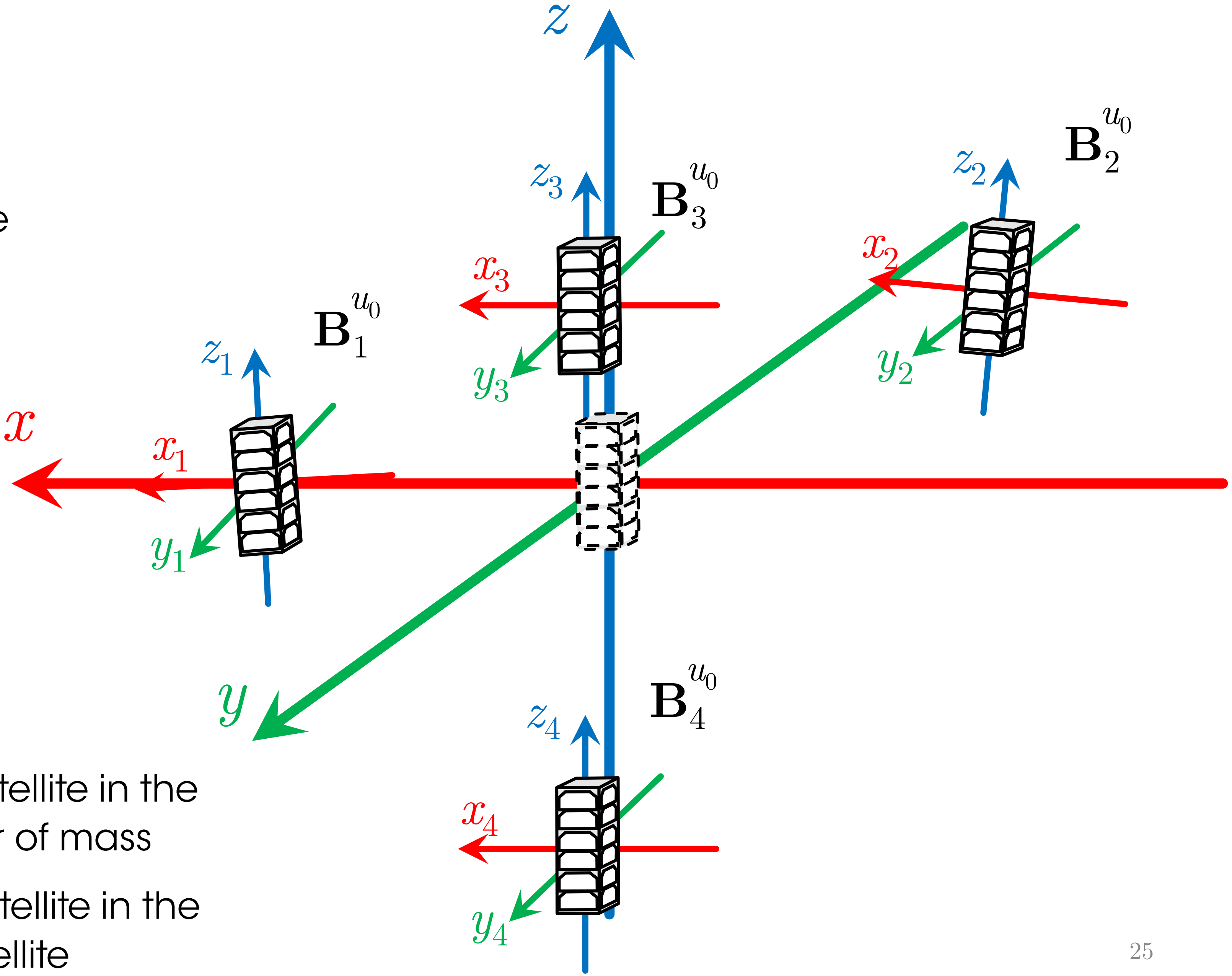
$\mathbf{B}_j^{q_j}$ – magnetic field of j[th] satellite in the **body-fixed** frame of j[th] satellite

$\mathbf{B}_j^{u_j}$ – magnetic field of j[th] satellite in the **orbital** frame of j[th] satellite

Data exchange

With known DCM $\mathbf{A}^{u}_{j\rightarrow 0}$,
transform measurement to the orbital frame center of mass:

$$\mathbf{B}^{u_0}_{j} = \mathbf{A}^{u}_{j\rightarrow 0}\,\mathbf{B}^{u_j}_{j}$$

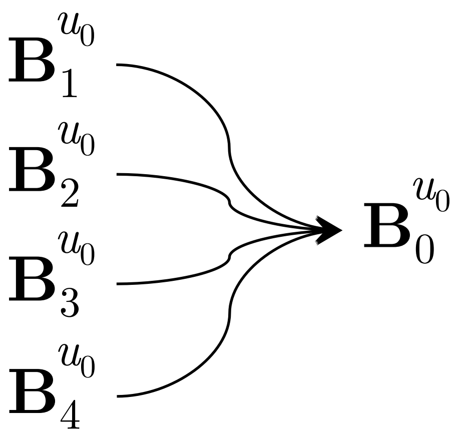


where:

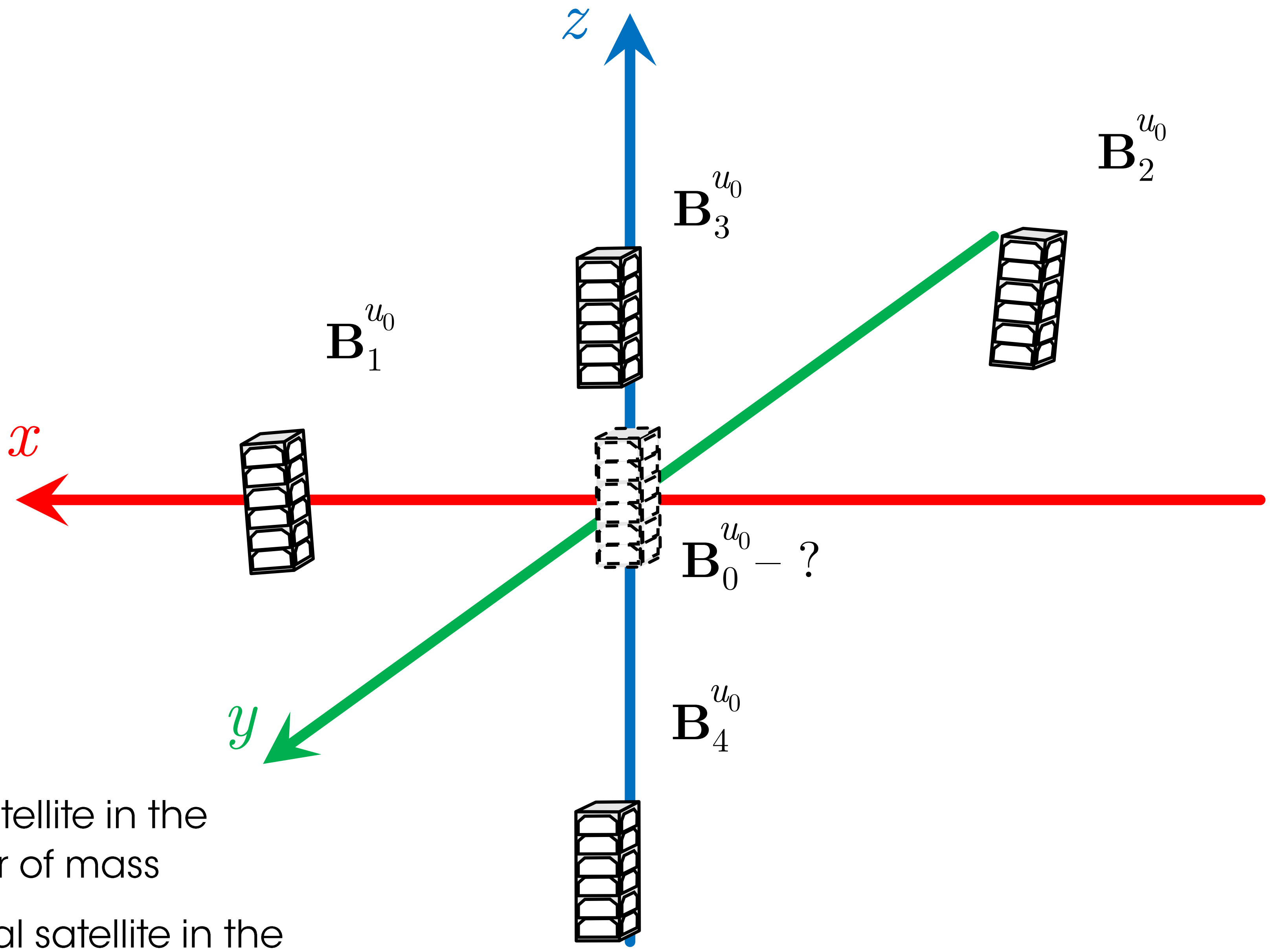
$\mathbf{B}^{u_0}_{j}$ – magnetic field of j[th] satellite in the **orbital** frame of center of mass

$\mathbf{B}^{u_j}_{j}$ – magnetic field of j[th] satellite in the **orbital** frame of j[th] satellite

Interpolation



Kriging

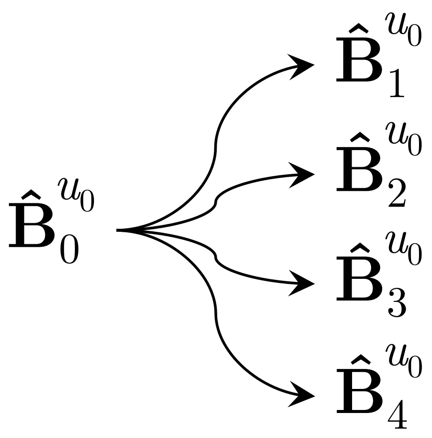


where:

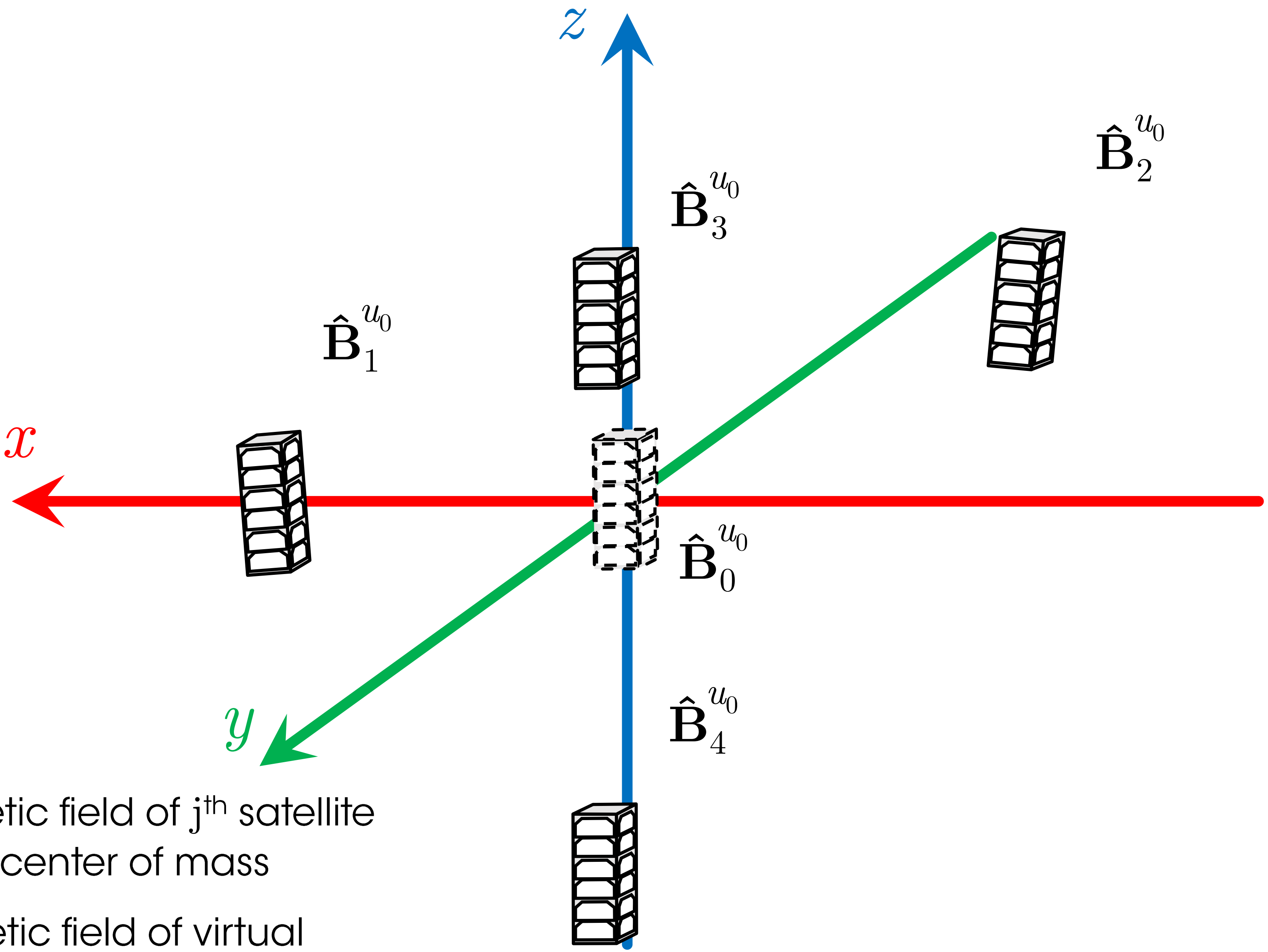
$\mathbf{B}_j^{u_0}$ – magnetic field of j-th satellite in the **orbital** frame of center of mass

$\mathbf{B}_0^{u_0}$ – magnetic field of virtual satellite in the **orbital** frame of center of mass

Extrapolation



using direct dipole structure



where:

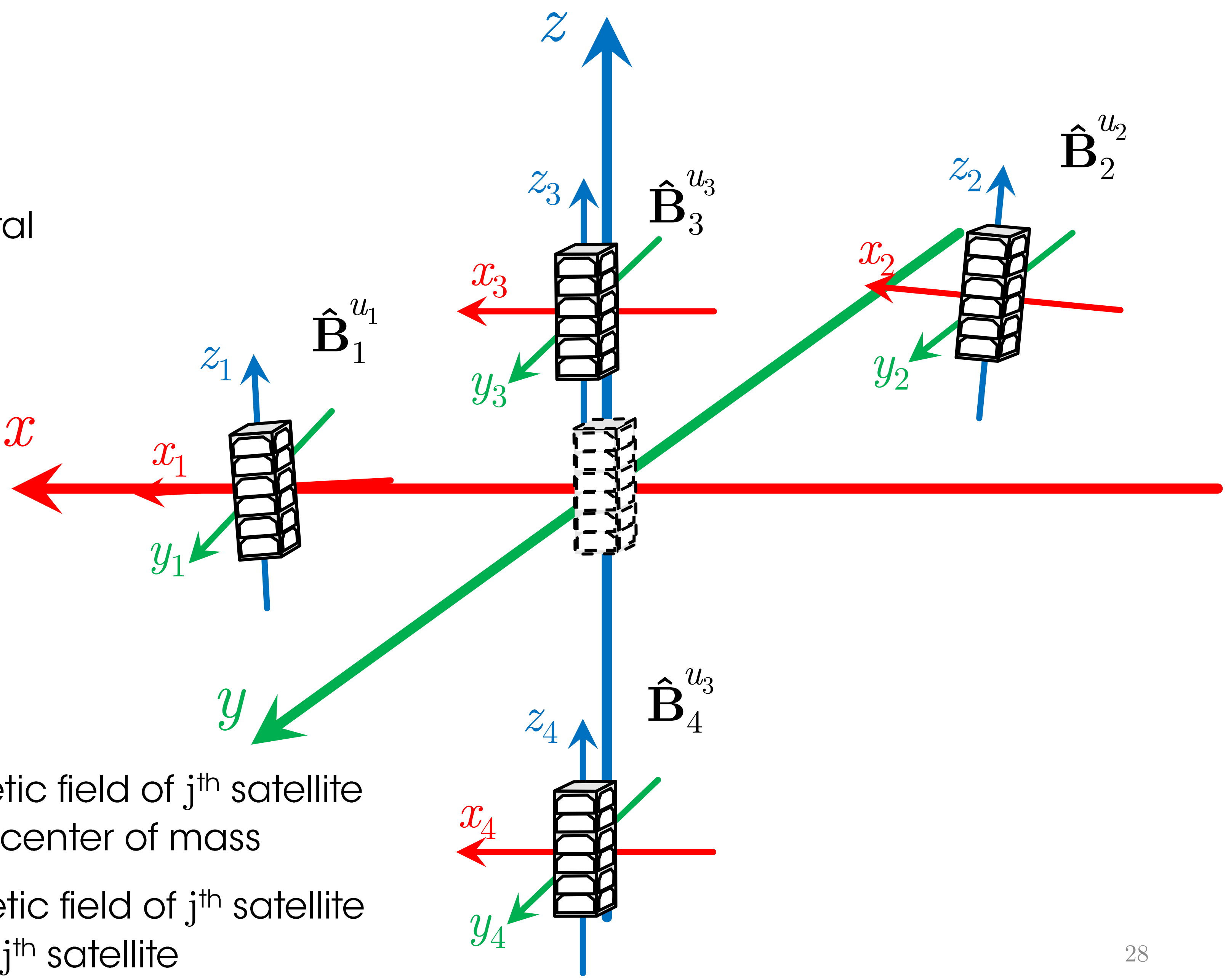
$\hat{\mathbf{B}}_j^{u_0}$ – estimate of the magnetic field of j[th] satellite in the **orbital** frame of center of mass

$\hat{\mathbf{B}}_0^{u_0}$ – estimate of the magnetic field of virtual satellite in the **orbital** frame of center of mass

Extrapolation

With known DCM $\mathbf{A}^{u}_{0\to j}$, transform estimate to the orbital frame of j[th] satellite :

$$\hat{\mathbf{B}}^{u_j}_{j} = \mathbf{A}^{u}_{j\to 0}\, \hat{\mathbf{B}}^{u_0}_{j}$$



where:

$\hat{\mathbf{B}}^{u_0}_{j}$ – estimate of the magnetic field of j[th] satellite in the **orbital** frame of center of mass

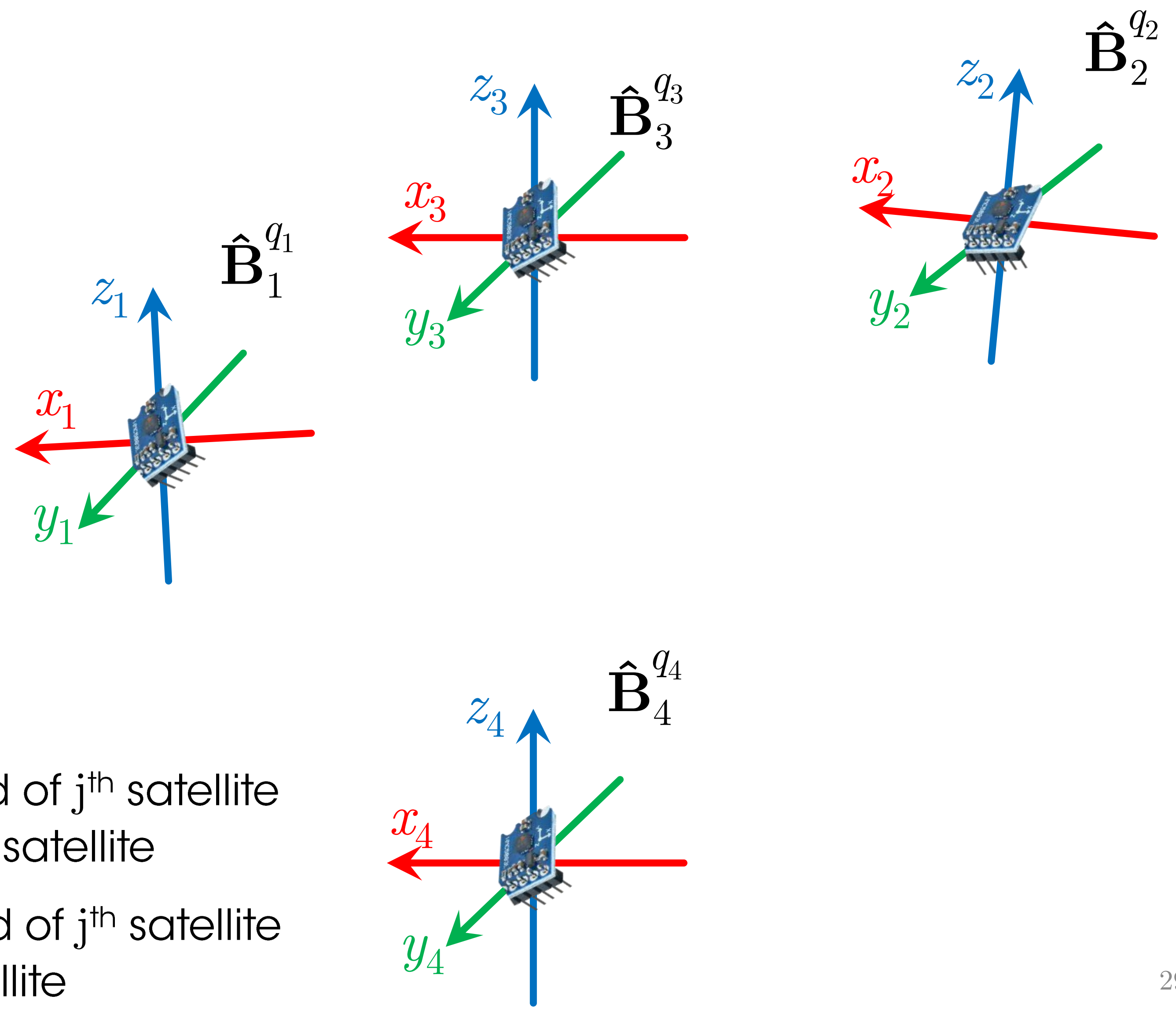
$\hat{\mathbf{B}}^{u_j}_{j}$ – estimate of the magnetic field of j[th] satellite in the **orbital** frame of j[th] satellite

Extrapolation

With known quaternion Q_j transform estimate to the body-fixed frame of j[th] satellite:

$$\hat{\mathbf{B}}_j^{q_j} = \tilde{Q}_j \circ \hat{\mathbf{B}}_j^{u_j} \circ Q_j$$

Enhanced estimates $\hat{\mathbf{B}}_j^{q_j}$ go to the observation model of EKF

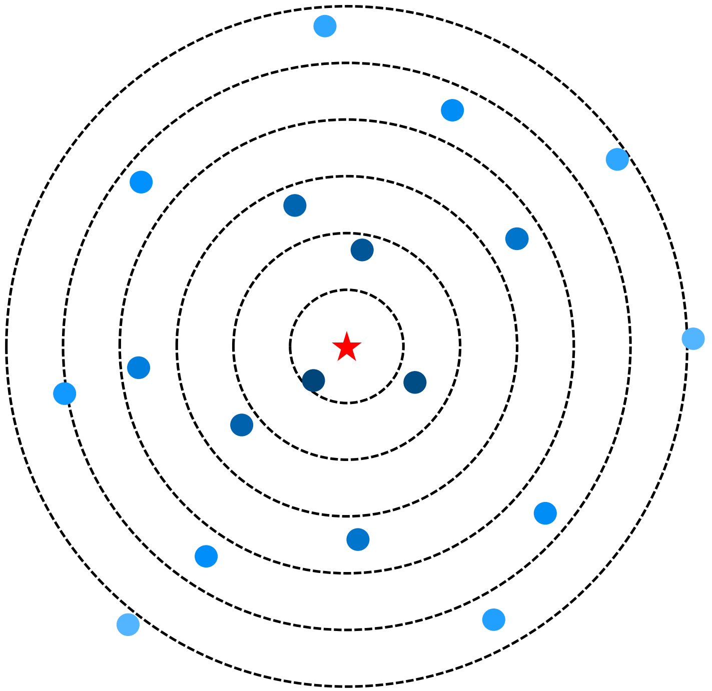


where:

- $\hat{\mathbf{B}}_j^{q_j}$ – estimate of the magnetic field of j[th] satellite in the **body-fixed** frame of j[th] satellite
- $\hat{\mathbf{B}}_j^{u_j}$ – estimate of the magnetic field of j[th] satellite in the **orbital** frame of j[th] satellite

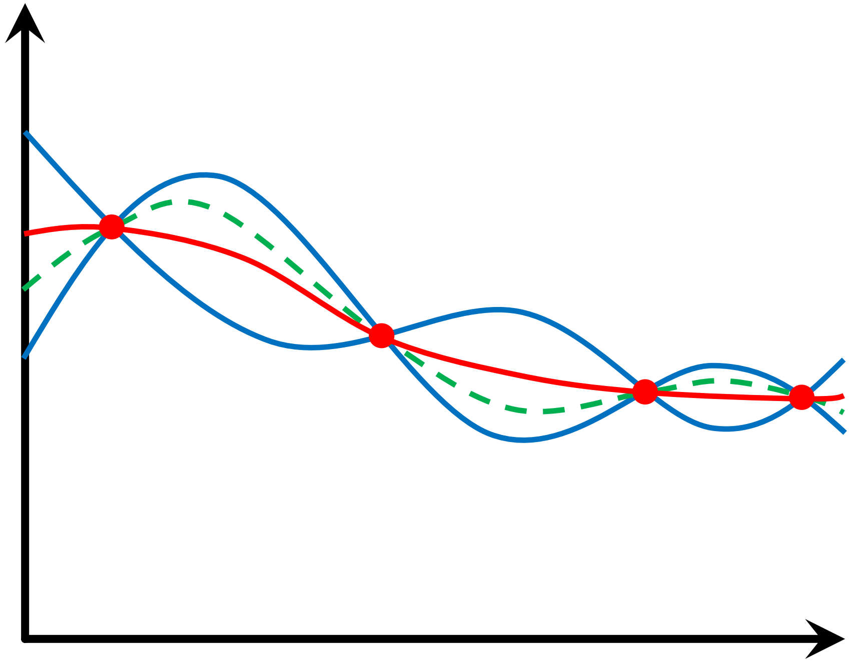
Interpolation

Inverse Distance Weighting



Splines

Kriging



Analytical method

Analytical method

Statistical method

Kriging

Estimator: $$\hat{\mathbf{B}}(\mathbf{R}_0) = \sum_{j=1}^{n} k_j \, \mathbf{B}(\mathbf{R}_j)$$

Notations:

- $\mathbf{R}_0$ — interpolated point
- $\mathbf{R}_j$ — available point with measurement
- k_j — weight (spatial correlation)
- $\hat{\mathbf{B}}$ — estimate (predicted value)
- $\mathbf{B}$ — measurement
- n — number of available points

Covariance

Estimator: $$\hat{\mathbf{B}}(\mathbf{R}_0) = \sum_{j=1}^{n} k_j\, \mathbf{B}(\mathbf{R}_j)$$

Translation invariance

of mean: $$\forall \mathbf{R} : \ \mathbf{E}\big[\mathbf{B}(\mathbf{R})\big] = \mathrm{const}$$

of covariance: $$\forall \mathbf{R}_i, \mathbf{R}_j : \ |\mathbf{R}_i - \mathbf{R}_j| = h$$

$$\mathrm{cov}(\mathbf{R}_i, \mathbf{R}_j) = \mathrm{C}(\mathbf{R}_i - \mathbf{R}_j) = \mathrm{C}(h)$$

Ordinary Kriging

Notations:

- $\mathbf{R}_0$ — interpolated point
- $\mathbf{R}_j$ — available point with measurement
- k_j — weight (spatial correlation)
- $\hat{\mathbf{B}}$ — estimate (predicted value)
- $\mathbf{B}$ — measurement
- n — number of available points
- $\mathbf{E}$ — expected value (mean)
- h — distance between points (lag)
- C — translation-invariant covariance

Equation on coefficients

Estimator:

$$\hat{\mathbf{B}}(\mathbf{R}_0)=\sum_{j=1}^{n} k_j\, \mathbf{B}(\mathbf{R}_j)$$

Unbiasedness:

$$\mathbf{E}\left[\hat{\mathbf{B}}(\mathbf{R}_0)\right] = \mathbf{E}\left[\mathbf{B}(\mathbf{R}_0)\right]$$

$$\downarrow$$

$$\sum_{j=1}^{n} k_j = 1$$

Minimal variance:

$$\min\ \mathbf{D}\left[\hat{\mathbf{B}}(\mathbf{R}_0) - \mathbf{B}(\mathbf{R}_0)\right]$$

$$\downarrow$$

$$\sum_{j=1}^{n} k_j\, \mathrm{C}(\mathbf{R}_i - \mathbf{R}_j) - \varkappa = \mathrm{C}(\mathbf{R}_i - \mathbf{R}_0)$$

Notations:

- $\mathbf{R}_0$ — interpolated point
- $\mathbf{R}_j$ — available point with measurement
- k_j — weight (spatial correlation)
- $\hat{\mathbf{B}}$ — estimate (predicted value)
- $\mathbf{B}$ — measurement
- n — number of available points
- $\mathbf{D}$ — variance (dispersion)
- $\varkappa$ — Lagrange multiplier
- C — translation-invariant covariance

Semivariance

Estimator:

$$\hat{\mathbf{B}}(\mathbf{R}_0) = \sum_{j=1}^{n} k_j \, \mathbf{B}(\mathbf{R}_j)$$

Semivariance:

$$\gamma(\mathbf{R}_i - \mathbf{R}_j) = \frac{1}{2} \mathbf{D} \left[\mathbf{B}(\mathbf{R}_i) - \mathbf{B}(\mathbf{R}_j) \right] =$$

$$= \mathrm{C}(0) - \mathrm{C}(\mathbf{R}_i - \mathbf{R}_j)$$

Equations on coefficients:

n+1 equations

$$\begin{cases} \sum_{j=1}^{n} k_j = 1 \\ \sum_{j=1}^{n} k_j \, \gamma(\mathbf{R}_i - \mathbf{R}_j) + \varkappa = \gamma(\mathbf{R}_i - \mathbf{R}_0) \end{cases}$$

Notations:

- $\mathbf{R}_0$ — interpolated point
- $\mathbf{R}_j$ — available point with measurement
- k_j — weight (spatial correlation)
- $\hat{\mathbf{B}}$ — estimate (predicted value)
- $\mathbf{B}$ — measurement
- n — number of available points
- γ — semivariance
- $\varkappa$ — Lagrange multiplier
- C — translation-invariant covariance

Semivariogram

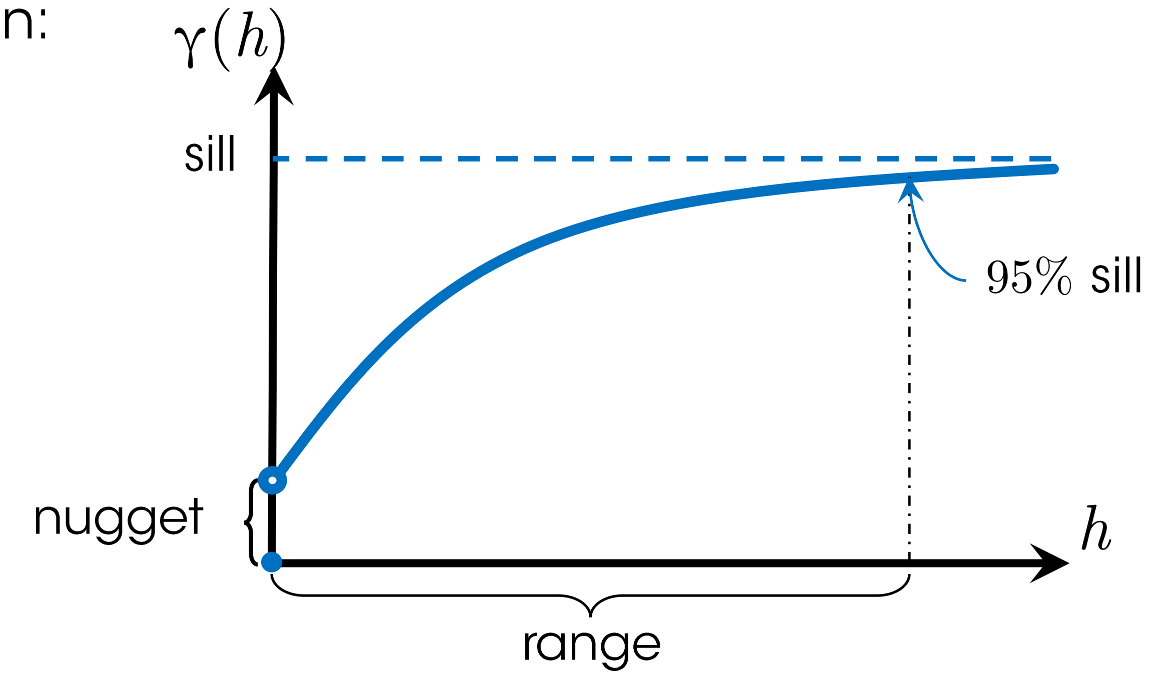
Empirical:

$$\forall \mathbf{R}_i, \mathbf{R}_j : \ |\mathbf{R}_i - \mathbf{R}_j| = h$$

$$\gamma(h) = \frac{1}{2n_h} \sum_{(i,j)=1}^{n_h} (\mathbf{B}(\mathbf{R}_i) - \mathbf{B}(\mathbf{R}_j))^2$$

n_h – total number of sampled points

Model function:



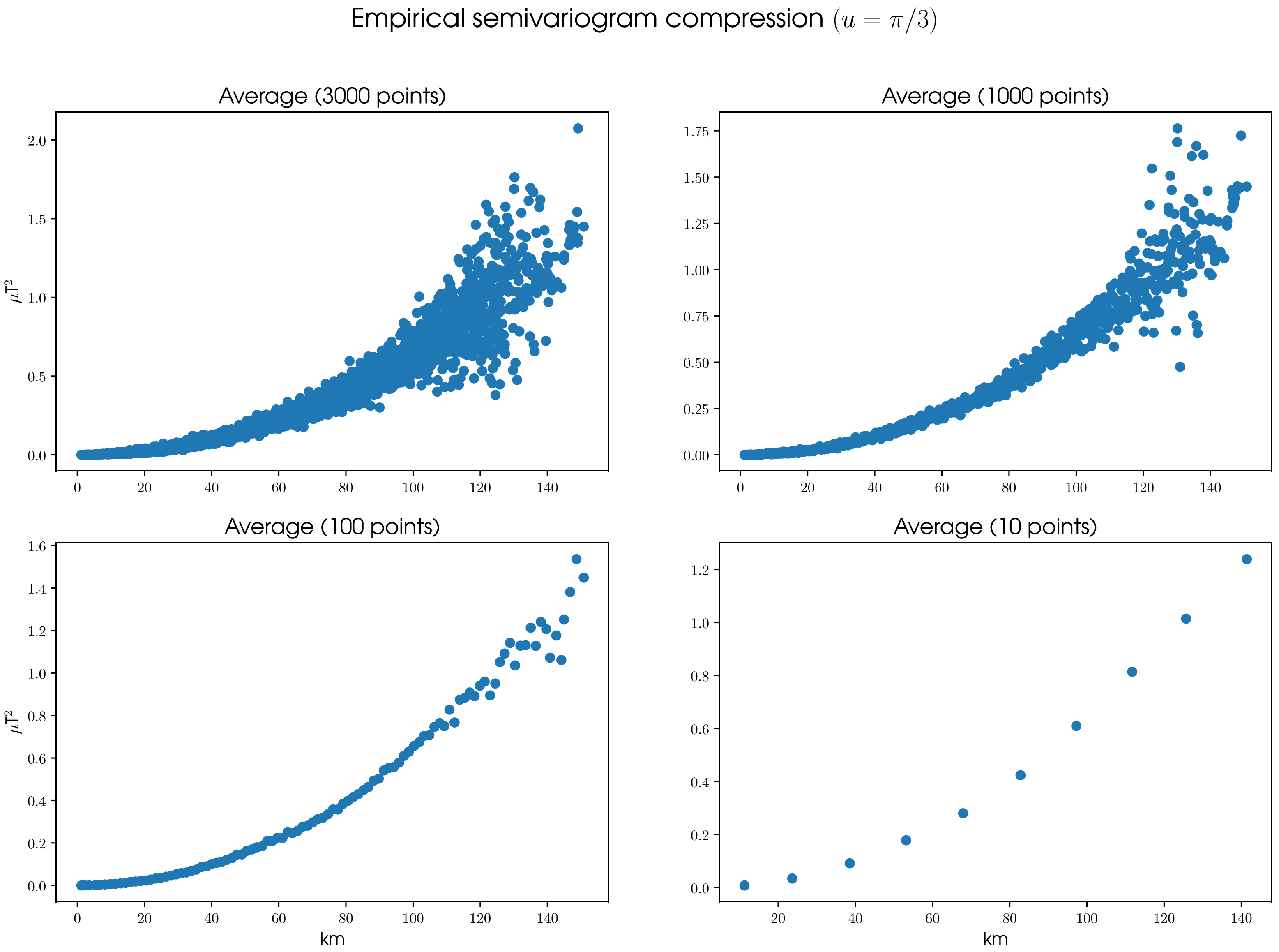
should be fit into empirical cloud of points

Semivariogram for direct dipole

Region:

Circular orbit with $i = 60^o$ and $h_{\text{orb}} = 750\ \text{km}$

Cube with size $50\ \text{km}$ with the center on this orbit (point $u = 60^o$)



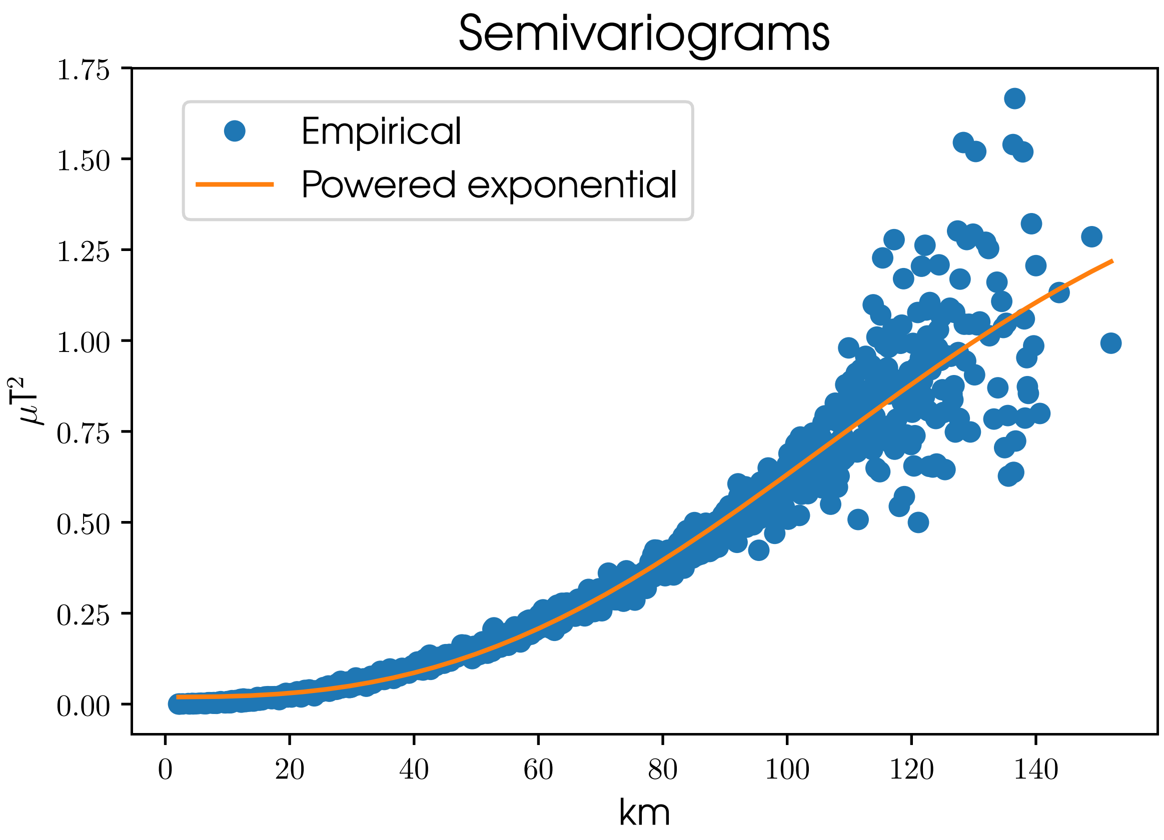
Semivariogram for direct dipole

Powered exponential model function:

$$\gamma(h) = \begin{cases} c_0 + c\left(1 - \exp\left[-\left(\frac{h}{a}\right)^{\nu}\right]\right), & h > 0 \\ 0\ , & h = 0 \end{cases}$$

After fitting:

$$\begin{aligned} c_0 &= 0.02\ \mu\text{T}^2 \\ c &= 1.5\ \mu\text{T}^2 \\ a &= 127\ \text{km} \\ \nu &= 8/3 \end{aligned}$$



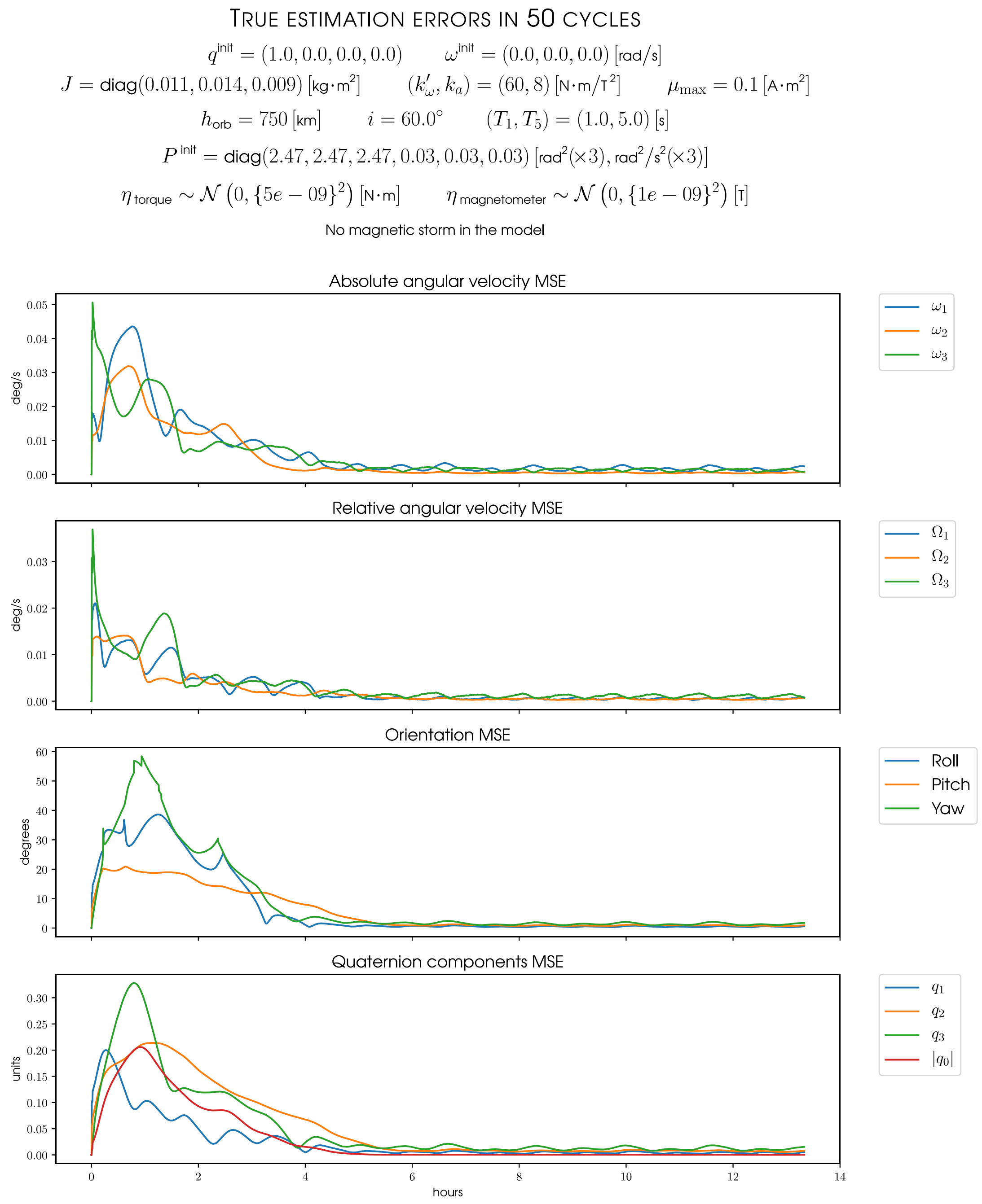
True estimation errors

$$\sigma_{\text{torque}} = 5 \text{ nN·m}$$

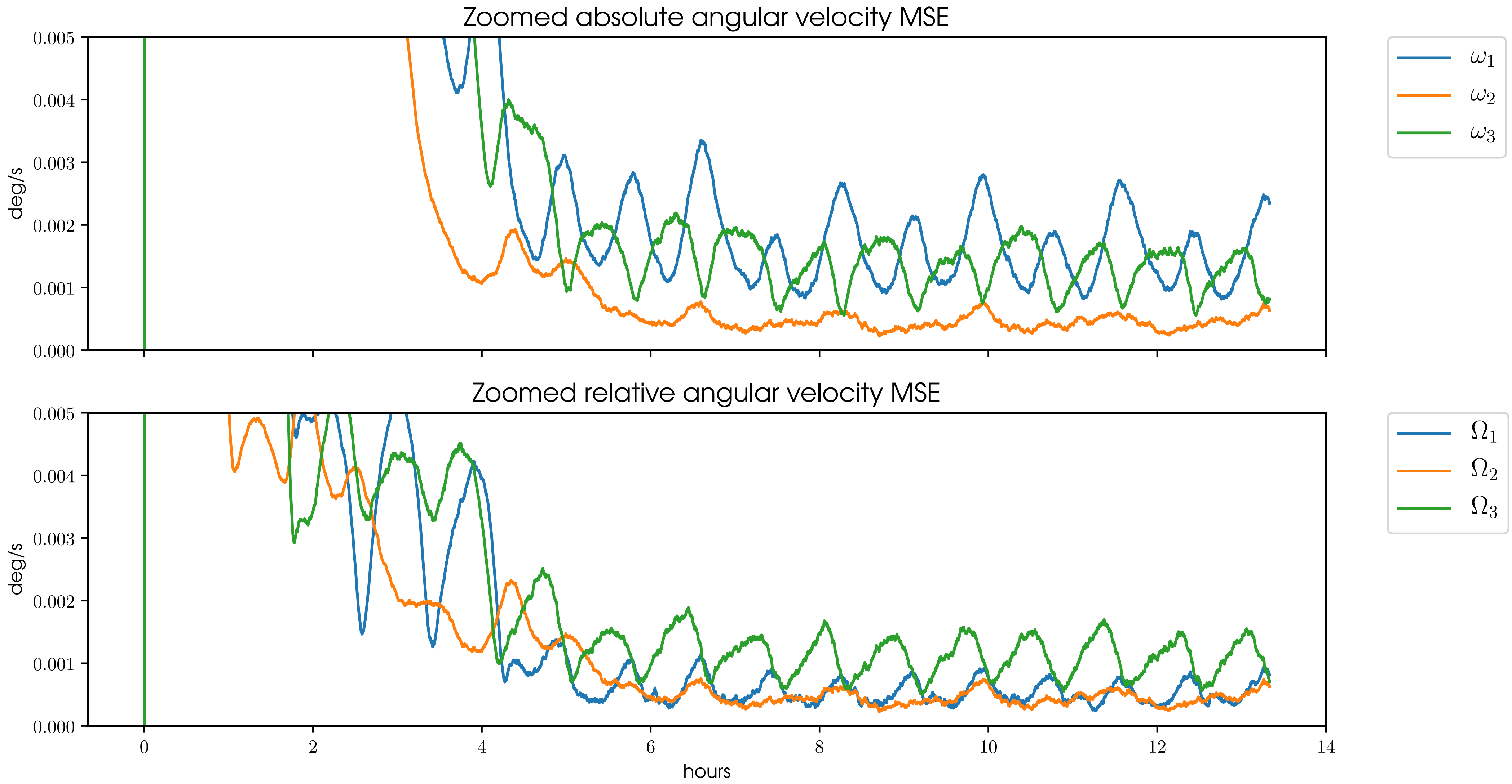
$$\sigma_{\text{meas}} = 1 \text{ nT}$$

$$\mathbf{B}_{\text{bias}} = 0$$

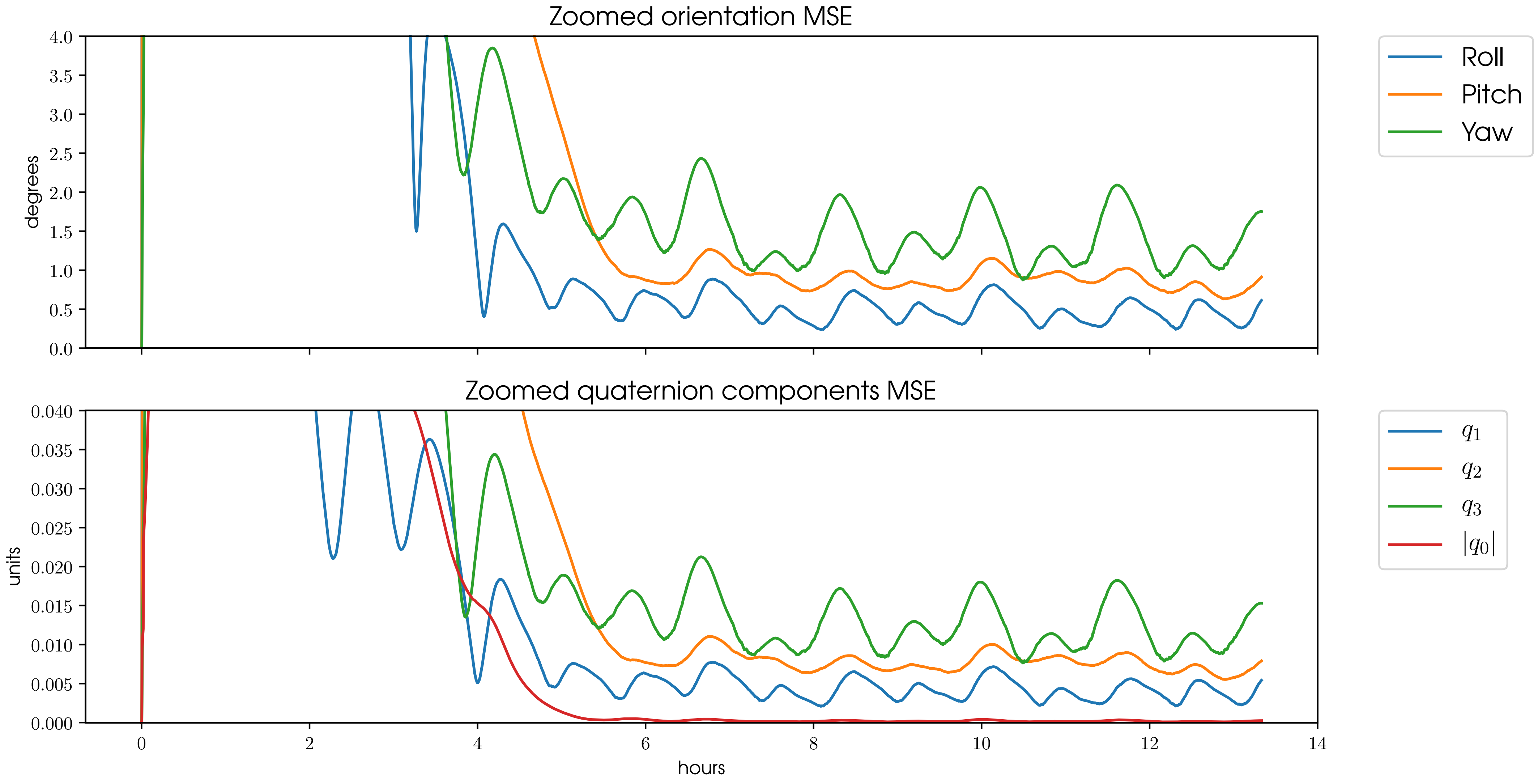
$$\sigma_{\text{artificial}} = 1 \text{ nT}$$



Angular velocities MSE



Orientation MSE



Conclusions

- Filtration of CubeSat's magnetic ADCS is performed and compared with filtration in swarm
- Interpolation technique Kriging is implemented
- Empirical and analytical semivariograms for direct dipole magnetic field are acquired
- Filtration with interpolation gives on average 0.5 degrees better accuracy

Future plans

- Research on the impact of different semivariograms
- Research on the actual range of Kriging
- To get rid of swarm's rigid body movement
- Non-ordinary Kriging
- Different parameters for empirical semivariograms
- Research on the number of CubeSats in swarm
- IGRF magnetic field model
- Research on control and measurement cycle, several measurements in a row

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Thank you for your attention!