Center of mass compensation of a nanosatellite testbed based on the Extended Kalman Filter

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Overview

Testbed characteristics

Technical specifications

- **Hardware**
  - IMU (3-axis mag, gyro and accel)
  - 3 Movable Mass Units
  - Embedded μcontroller
  - 3 Reaction wheels
- **Structure**
  - Air bearing
  - Helmholtz cage

Figure: LAICA nanosatellite testbed.

The balancing problem

Some facts

- Although limited in the roll and pitch axis to $\pm 45°$, **the testbed has 3 rotational Degrees of Freedom (DOF)**;
- Considering the testbed is not a symmetrical rigid body, its mass distribution results in a **Center of Mass (CM)** displaced from its **Center of Rotation (CR)**;
- This displacement results in a undesirable torque which affects the **actuation system**.

Solution?

Discovering the exact position of the Center of Mass (CM) and controlling its position by moving some masses on the testbed.

The balancing problem

How is it done?

In this work, in order to discover the position of the Center of Mass (CM), the testbed is excited with the torque generated by the reaction wheels mounted on it. Then, with the knowledge of this input torque and measuring the angular velocities performed by the testbed during this operation, the Extended Kalman Filter (EKF) is used in order to estimate the testbed parameters given by **R**, the CM offset vector, and **J**, the inertia tensor given by a 3×3 matrix.

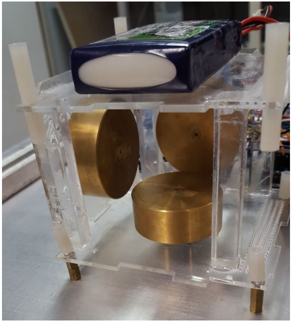


Figure: 3-axis setup of reaction wheels

Rigid body dynamics

Euler equation

$$\frac{d\mathbf{H}}{dt} = \mathbf{J}\dot{\omega} + \omega \times \mathbf{J}\omega + (\dot{\mathbf{h}} + \omega \times \mathbf{h}) = \mathbf{R} \times m\mathbf{g_b} + \mathbf{T} \ , \tag{1}$$

- $\mathbf{J}$ - the inertia tensor of the whole system
- $\mathbf{H}$ - total angular momentum of the system
- $\mathbf{h}$ - momentum generated by the reaction wheels
- m - the total mass of the system
- $\mathbf{T}$ - the total external torque disturbances
- $\mathbf{R}$ - the CM offset vector
- $\mathbf{g_b}$ - the local gravity (in body frame)

Kinematic equation

Euler rates for the ZYX (or 3-2-1) rotation sequence

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & sin(\phi) \cdot tan(\theta) & cos(\phi) \cdot tan(\theta) \\ 0 & cos(\phi) & -sin(\theta) \\ 0 & sin(\phi)/cos(\theta) & cos(\phi)/cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

- ϕ - roll angle of the testbed
- θ - pitch angle of the testbed
- ψ - yaw angle of the testbed
- ω_x - angular velocity of the testbed in the body x-axis
- ω_y - angular velocity of the testbed in the body y-axis
- ω_z - angular velocity of the testbed in the body z-axis

Augmented State Equation

The $\mathbf{x}$ state vector

- For the EKF algorithm, the state of the system is selected as being $\mathbf{x_1} = [\omega_x \ \omega_y \ \omega_z]^T$.
- Using the joint state equation procedure, the state is augmented to $\mathbf{x} = [\mathbf{x_1} \ \mathbf{x_2}]^T$, where $\mathbf{x_2} = [J_x \ J_y \ J_z \ J_{xy} \ J_{xz} \ J_{yz} \ mR_x \ mR_y \ mR_z]^T$.
- Since $\mathbf{x_2}$ contains the parameters to be estimated, which are constant, the joint state equation becomes:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{x_1}} \\ \dot{\mathbf{x_2}} \end{bmatrix} = F(\mathbf{x_1}, \mathbf{x_2}, \mathbf{u}) = \begin{bmatrix} f(\mathbf{x_1}, \mathbf{x_2}, \mathbf{u}) \\ 0 \end{bmatrix}, \tag{3}$$

where $f(\mathbf{x_1}, \mathbf{x_2}, \mathbf{u}) = \dot{\omega} = J^{-1}(J\omega \times \omega + R \times m\mathbf{g} - (\dot{\mathbf{h}} + \omega \times \mathbf{h}))$ and $\mathbf{u} = -(\dot{\mathbf{h}} + \omega \times \mathbf{h})$.

Rotation Sequence

The ZYX or 321 sequence

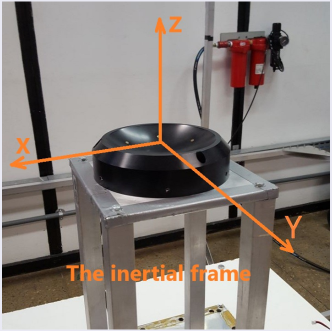


Figure: Inertial frame (centered at the CR of the Air Bearing).

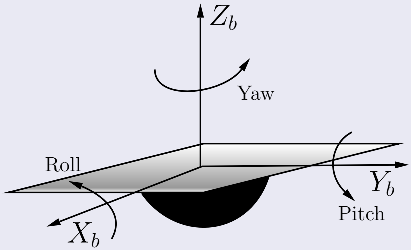


Figure: Body frame (centered at the CR of the Air Bearing, but fixed to the testbed).

Inertia tensor, mass and unbalance vector

Table: Testbed parameters used in the simulation.

J $(kg \cdot m^2)$[1]	Mass (kg)	Unbalance vector[2] $(10^{-3} m)$
$\begin{bmatrix} 0.265 & 0 & 0 \\ 0 & 0.246 & 0 \\ 0 & 0 & 0.427 \end{bmatrix}$	14.307	$\begin{bmatrix} -1 & -1 & -5 \end{bmatrix}^T$

Table: Standard deviation of the noise in the gyro readings

Body axis	Standard deviation (rad/s)
X	0.0026
Y	0.0049
Z	0.0031

[1] The Inertia Tensor (J) and the Mass (m) of the platform values in the table are the same as those estimated for the actual testbed.

[2] The shown values for the unbalance vector components were arbitrarily assumed for the simulation (a reasonable value based on previous experiments).

Simulation of the dynamic and kinematic equations

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{A}(\phi, \theta, \psi)_{\mathbf{3 \times 3}} \cdot \mathbf{x_1} \quad (4)$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_{new} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_{old} + \mathbf{A}(\phi, \theta, \psi)_{\mathbf{3 \times 3}} \cdot \mathbf{x_1} \cdot dt \quad (5)$$

$$\dot{\mathbf{x_1}} = J^{-1}(J\mathbf{x_1} \times \mathbf{x_1} + R \times m\mathbf{g} - (\dot{\mathbf{h}} + \mathbf{x_1} \times \mathbf{h})) \quad (6)$$

$$\mathbf{x}_{new} = \mathbf{x}_{old} + \dot{\mathbf{x}} \cdot dt \quad (7)$$

$$z = H \cdot x + w, \quad (8)$$

where w is the measurement noise, dt is the step size, which was selected as $0.0001s$ and the measurement matrix H is given by $H = [I_{3\times 3} \; 0_{3\times 9}]^T$.

Execution of the filter equations

$$F = \left.\frac{D\mathbf{F}}{D\mathbf{x}}\right|_{\hat{\mathbf{x}}} + [I]_{12\times12} \tag{9}$$

$$\dot{P} = F \cdot P + P \cdot F^T + L \cdot Q \cdot L^T - P \cdot H^T / R \cdot H \cdot P \tag{10}$$

$$P_{new} = P_{old} + \dot{P} \cdot dt \tag{11}$$

$$K = P \cdot H^T \cdot R^{-1} \tag{12}$$

$$\dot{\hat{x}} = \begin{bmatrix} \hat{\mathbf{x_1}} \\ \mathbf{0}_{6\times1} \\ \mathbf{0}_{3\times1} \end{bmatrix} \tag{13}$$

$$\dot{\hat{x}}_{new} = \dot{\hat{x}}_{old} + K \cdot (z - H \cdot \dot{\hat{x}}) \tag{14}$$

$$\hat{\mathbf{x}}_{new} = \hat{\mathbf{x}}_{old} + \dot{\hat{x}} \cdot dt \tag{15}$$

The $\frac{DF}{Dx}$ jacobian

The $\frac{D\mathbf{F}}{D\mathbf{x}}$ jacobian is a 12×12 matrix given by

$$\frac{D\mathbf{F}}{D\mathbf{x}} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{12}}{\partial x_1} & \cdots & \frac{\partial F_{12}}{\partial x_{12}} \end{bmatrix} = [A_{ij}]_{12\times 12} = \left[\frac{\partial F_i}{\partial x_j}\right]_{12\times 12}, \quad i, j \in \{1, ..., 12\} \tag{16}$$

and, after simplification,

$$\left.\frac{D\mathbf{F}}{D\mathbf{x}}\right|_{\hat{\mathbf{x}}} = \left.\begin{bmatrix} \frac{\partial F}{\partial \omega} & \frac{\partial F}{\partial \mathbf{J}} & \frac{\partial F}{\partial \ m\mathbf{R}} \\ & \mathbf{0}_{9\times 12} & \end{bmatrix}\right|_{\hat{\mathbf{x}}}, \tag{17}$$

where $\frac{\partial F}{\partial \omega}$, $\frac{\partial F}{\partial \mathbf{J}}$ and $\frac{\partial F}{\partial \ m\mathbf{R}}$ are 3×3, 3×6 and 3×3 matrices, respectively.

Complex Step Differentiation idea for the $\frac{DF}{dx}$ calculation

The numerical differentiation problem

One simple discrete method for numerical differentiation is given by

$$\frac{df}{dx} \approx \frac{f(x+h) - f(x)}{h}, \quad \textit{where } h \textit{ is the step size.} \tag{18}$$

However, when the step size h becomes too small, the procedure results in erratic values, mainly as consequence of subtraction errors.

Solution? Complex step differentiation!

Through complex step differentiation, the same differentiation may be accomplished using the approximation

$$\frac{df}{dx} \approx \frac{Im[f(x+ih)]}{h}, \quad \textit{where } i \textit{ is given by } i^2 = -1. \tag{19}$$

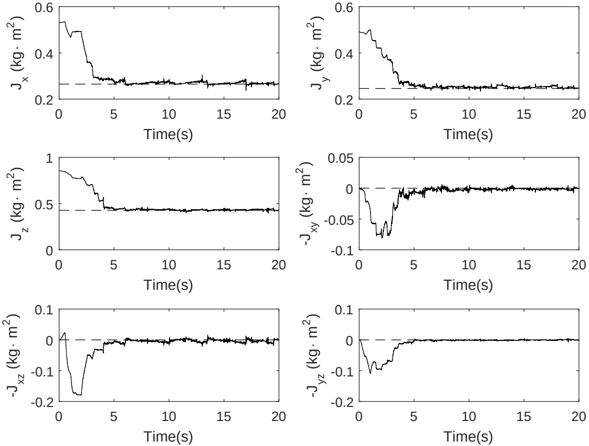
Simulation and initial parameters

The procedure for running the EKF is given by the following steps:

1. The simulator is configured with the testbed known parameters (the ground truth parameters).
2. The dynamic model is simulated. The torque generated by the reaction wheels and initial conditions of the testbed are given as **input**. The simulator **outputs** the angular velocities of the testbed.
3. Noises are added to these angular velocities signals to simulate real measurement conditions.
4. With only the knowledge of these input and output signals, the EKF algorithm is executed. The estimated $\hat{\mathbf{x}}_2$ vector is collected after the algorithm converges.
5. Estimated $\hat{\mathbf{x}}_2$ parameters are compared with the ground truth $\mathbf{x}_2$ parameters.

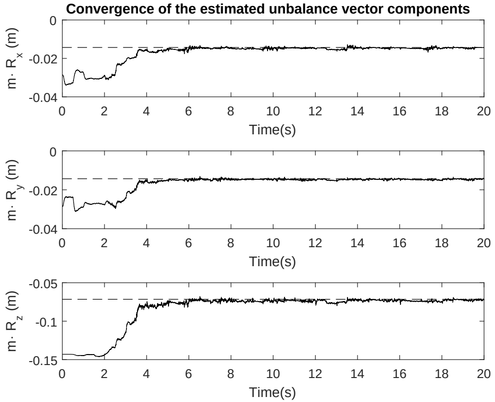
Results

Figure: Convergence of the estimated inertia tensor components.



Results

Figure: Unbalance vector components[3] (dashed = real, continuous = estimated).



[3]The graph shows the unbalance components scaled by the m factor (testbed mass).

Results

Within 5 secs the error between the known (ω_z) and the estimated ($\hat{\omega}_z$) angular velocities stays in the $\approx \pm 0.001 rad/s$ range.

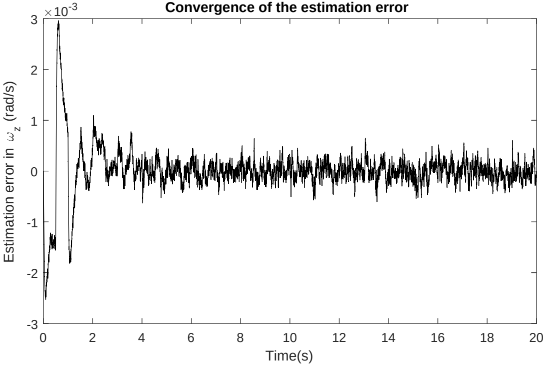


Figure: Evolution of the difference $\omega_z - \hat{\omega}_z$

Conclusions

1. This work presented a testbed being developed at the LAICA laboratory in the University of Brasília for testing attitude determination and control systems (ADCS) of nanosatellites, *e.g.* CubeSATs.
2. The balancing problem was presented, as well as the modelling for one solution for this problem, based on reaction wheel actuation and on the use of the Extended Kalman Filter.
3. The results shown indicate that the proposed method is adequate for solving the balancing problem.
4. Future works include discretizing the EKF and embedding it on the board computer of the real testbed.