Lyapunov based attitude control algorithm for slew maneuvers with restrictions

Yaroslav Mashtakov

Mikhail Ovchinnikov

Stepan Tkachev

Mark Shachkov

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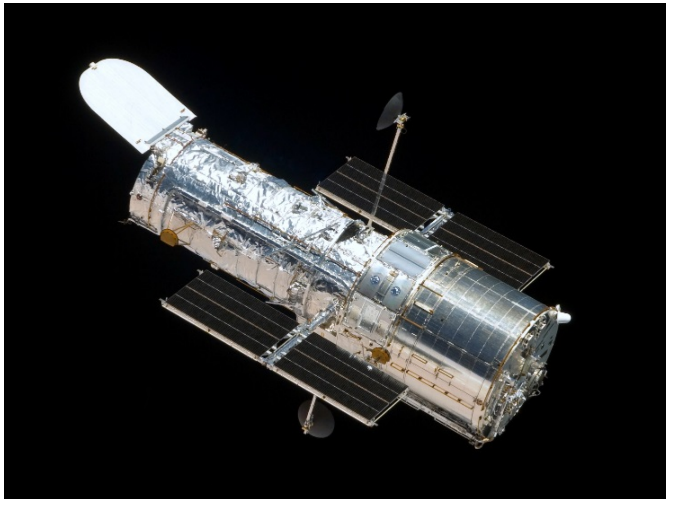
Introduction

We want to perform the slew maneuver

Restrictions:

- Camera axis does not look at bright objects
- Solar panels are directed to the Sun



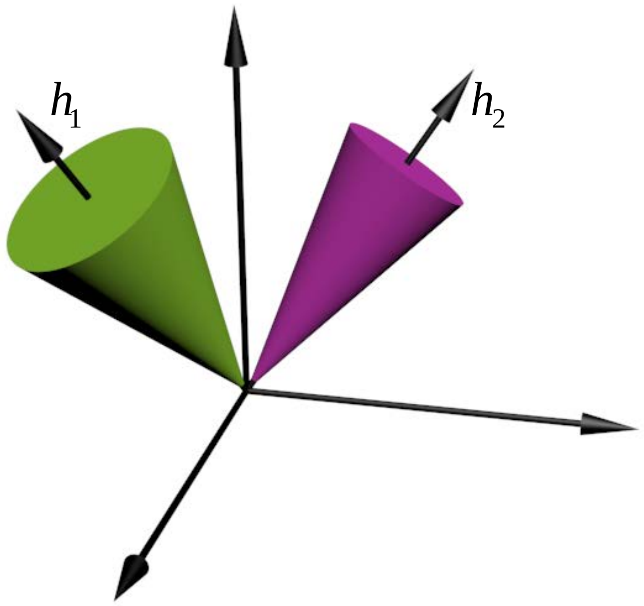
How to do that?

- Pontryagin's maximum principle
  - ✓Time optimal
  - ×Too complicated for on-board implementation
- Lyapunov based
  - ✓Simple
  - ✓Robust
  - ×Not time optimal

Problem statement

What do we know?

- Satellite parameters
- Forbidden areas (cones, do not move in Inertial space)
- Axis that should not be located in forbidden area (camera axis)
- Initial and reference attitude
- Initial and reference angular velocity equals zero



What do we want?

- Perform the slew maneuver
- Avoid the forbidden areas

Lyapunov based attitude control

- Widely used for attitude control
- Ensures asymptotic stability of the reference motion
- Robust

Main idea:

- Positive definite function
- Control ensures nonpositivity of its time derivative
- In accordance with Barbashin-Krasovskyy-LaSalle principle asymptotic stability is ensured

Examples

Function:

$$V_0 = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_a\left(3 - \mathrm{Tr}(A)\right)$$

Relative motion equations:

$$J\dot{\boldsymbol{\omega}}_{rel} + k_\omega \boldsymbol{\omega}_{rel} + k_a q_0 \mathbf{q} = 0$$

Control:

$$\mathbf{M}_{ctrl} = -\mathbf{M}_{ext} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} + \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_{ref} - \\ - \mathbf{J}\left[\boldsymbol{\omega}_{rel}\right]_\times \mathbf{A}\boldsymbol{\omega}_{ref} - k_a q_0 \mathbf{q} - k_\omega \boldsymbol{\omega}_{rel}$$

Function:

$$V_0 = \frac{1}{2}\left(\boldsymbol{\omega}_{rel}, \mathbf{J}\boldsymbol{\omega}_{rel}\right) + k_q\left(1 - q_0\right)$$

Relative motion equations:

$$J\dot{\boldsymbol{\omega}}_{rel} + k_\omega \boldsymbol{\omega}_{rel} + k_q \mathbf{q} = 0$$

Control:

$$\mathbf{M}_{ctrl} = -\mathbf{M}_{ext} + \boldsymbol{\omega}_{abs} \times \mathbf{J}\boldsymbol{\omega}_{abs} + \mathbf{J}\mathbf{A}\dot{\boldsymbol{\omega}}_{ref} - \\ - \mathbf{J}\left[\boldsymbol{\omega}_{rel}\right]_\times \mathbf{A}\boldsymbol{\omega}_{ref} - k_q \mathbf{q} - k_\omega \boldsymbol{\omega}_{rel}$$

$(q_0, \mathbf{q})$, A are quaternion and rotation matrix from Reference Frame to Body Frame

$\boldsymbol{\omega}_{rel}$ is relative angular velocity

$\mathbf{J}$ is tensor of inertia

k_a, k_ω, k_q are positive constants

Main idea

- Standard function V_0 cannot be applied
- Include restrictions: $V = V_0 \cdot f$
- f is a function that takes high values in the forbidden area
- Control ensures negativity of Lyapunov function time derivative, hence forbidden areas will be avoided
- Almost the same as the potential barrier around the forbidden area

Global function

$$f = \sum_{i=1}^{n} \frac{a_i}{\cos\alpha_i - (\mathbf{e},\mathbf{h}_i)}$$

a_i are constants, α_i is the minimum angle between the cone axis $\mathbf{h}_i$ and the camera axis $\mathbf{e}_f$

Lyapunov function:

$$V = \left(\frac{1}{2}(\boldsymbol{\omega},\mathbf{J}\boldsymbol{\omega}) + k_q(1-q_0)\right)\sum_{i=1}^{n} \frac{a_i}{\cos\alpha_i - (\mathbf{e},\mathbf{h}_i)}$$

Control (reference motion is inertial stabilization, n = 1 for simplicity):

$$\mathbf{M}_{ctrl} = \boldsymbol{\omega}\times J\boldsymbol{\omega} - \mathbf{M}_{ext} - k_q\mathbf{q} - k_\omega \frac{\cos\alpha - (\mathbf{e},\mathbf{h})}{a}\boldsymbol{\omega} - \mathbf{e}\times\mathbf{h}\frac{(\boldsymbol{\omega},\mathbf{J}\boldsymbol{\omega}) + 2k_q(1-q_0)}{\cos\alpha - (\mathbf{e},\mathbf{h})}$$

Local function

$$f_i = \begin{cases} 1, & (\mathbf{e}, \mathbf{h}_i) \le \cos\beta_i \\ a_i, & (\mathbf{e}, \mathbf{h}_i) \ge \cos\alpha_i \end{cases}, \quad \beta_i > \alpha_i$$

α_i is the minimum angle between the cone axis $\mathbf{h}_i$ and the camera axis $\mathbf{e}_f$

β_i is the angle where avoidance algorithm starts working

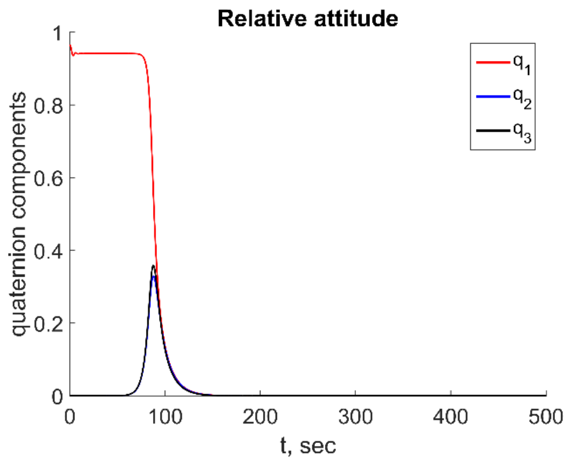
Lyapunov function:

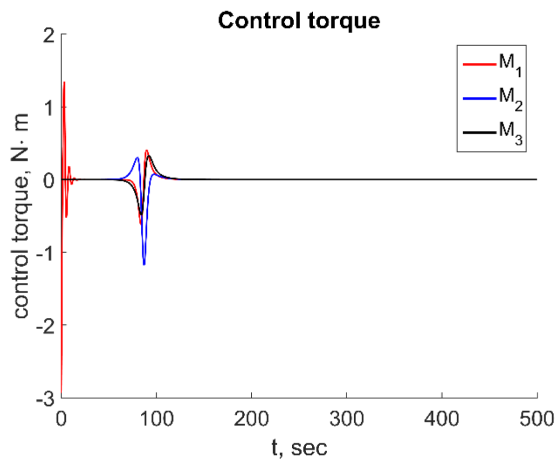
$$V = \left(\frac{1}{2}(\boldsymbol{\omega}, \mathbf{J}\boldsymbol{\omega}) + k_q(1 - q_0)\right)\prod_1^n f_i$$

Control (reference motion is inertial stabilization, n = 1 for simplicity):

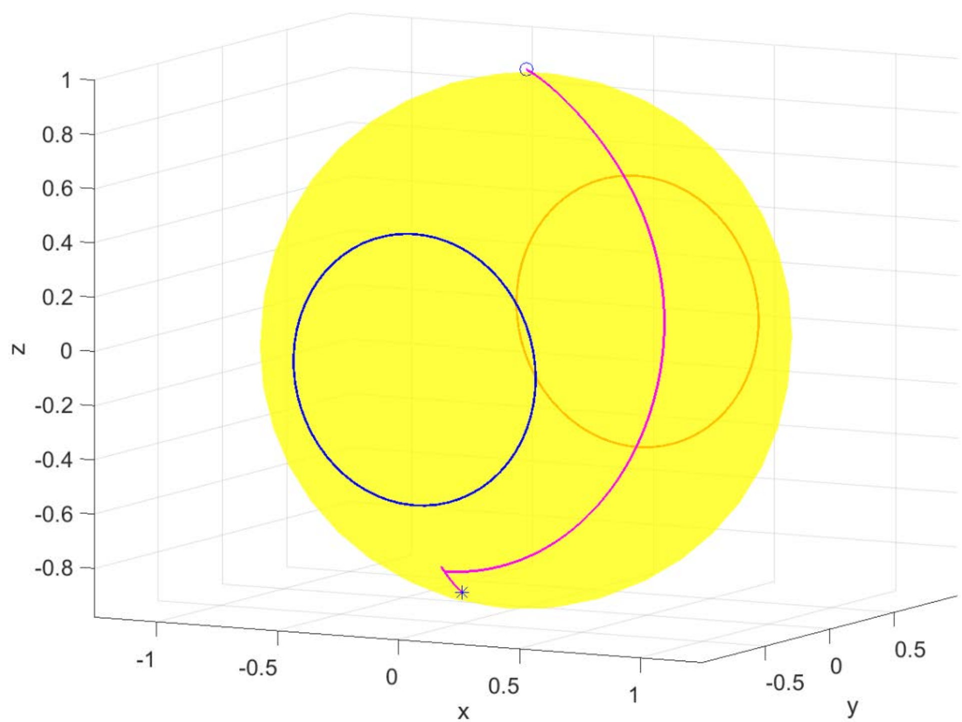
$$\mathbf{M}_{ctrl} = \boldsymbol{\omega} \times J\boldsymbol{\omega} - \mathbf{M}_{ext} - k_q\mathbf{q} - \frac{k_\omega}{f}\boldsymbol{\omega} - \mathbf{e} \times \mathbf{h}\left[\frac{1}{2}(\boldsymbol{\omega}, \mathbf{J}\boldsymbol{\omega}) + k_q(1 - q_0)\right]\frac{f'}{f}$$

Simulation results





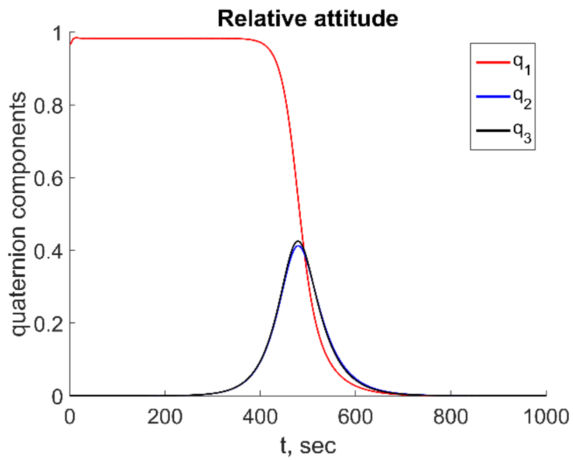
Global function

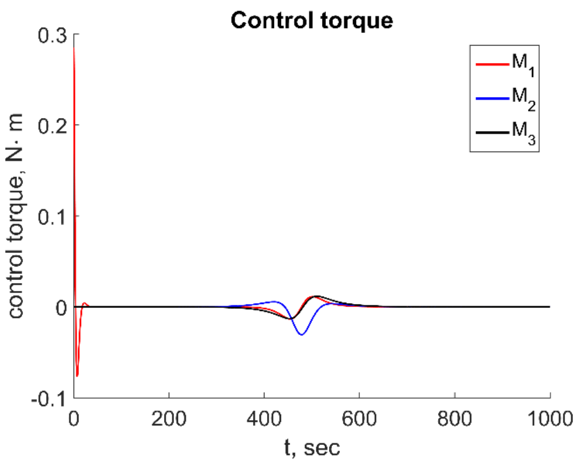


$$\mathbf{J} = \mathrm{diag}(20,\ 30,\ 40)$$

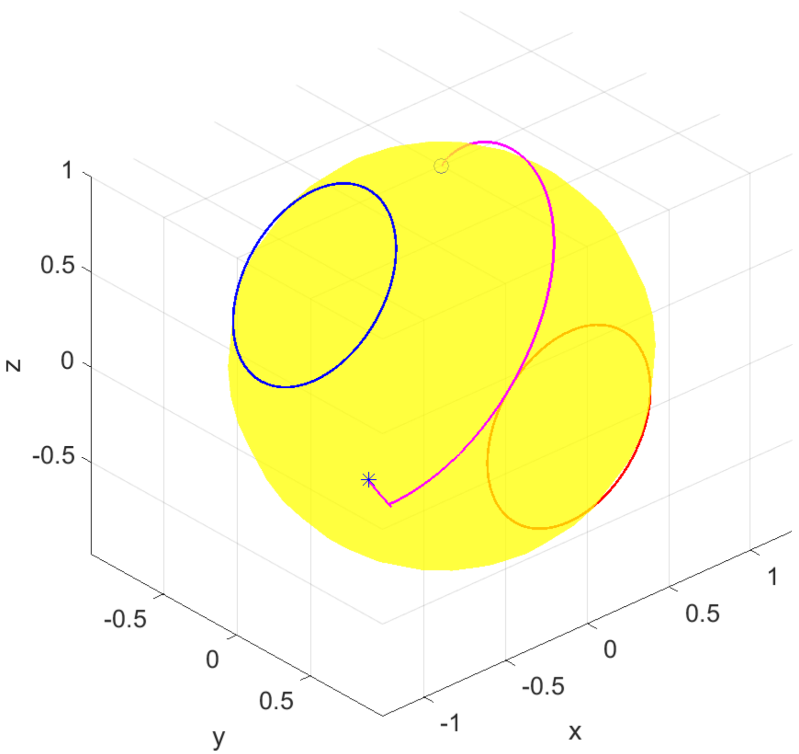
$$f = \sum_{i=1}^{2} \frac{0.05}{\cos\frac{\pi}{6} - (\mathbf{e}, \mathbf{h}_i)}$$

Simulation results



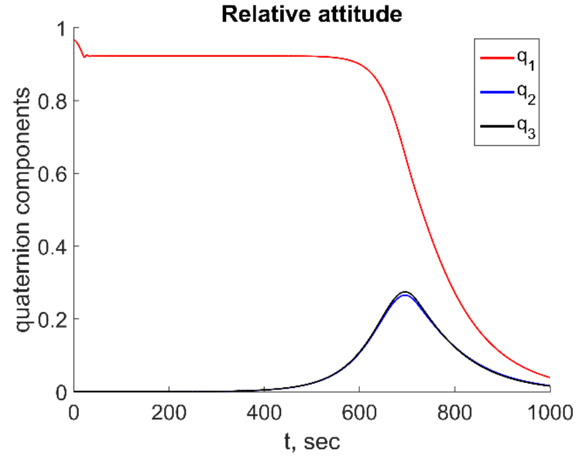


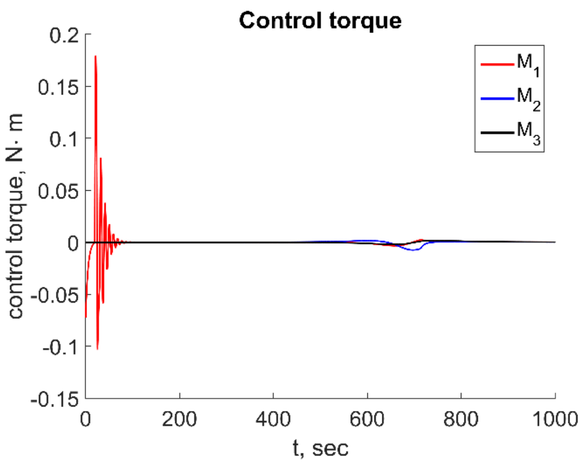
Global function



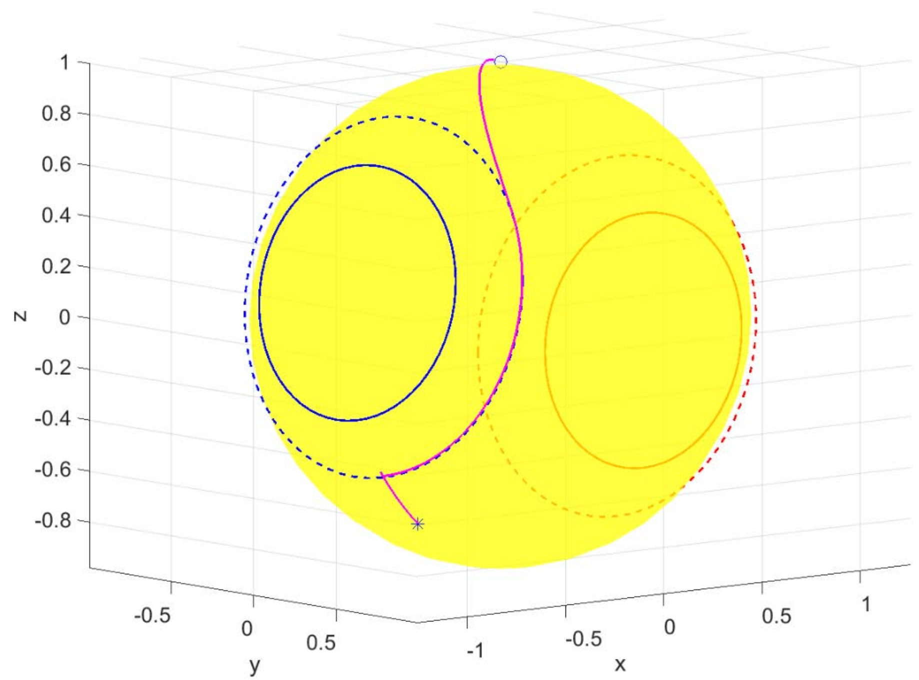
$$\mathbf{J} = \mathrm{diag}(20,\ 30,\ 40) \qquad f = \sum_{i=1}^{2} \frac{0.2}{\cos\frac{\pi}{6} - (\mathbf{e}, \mathbf{h}_i)}$$

Simulation results





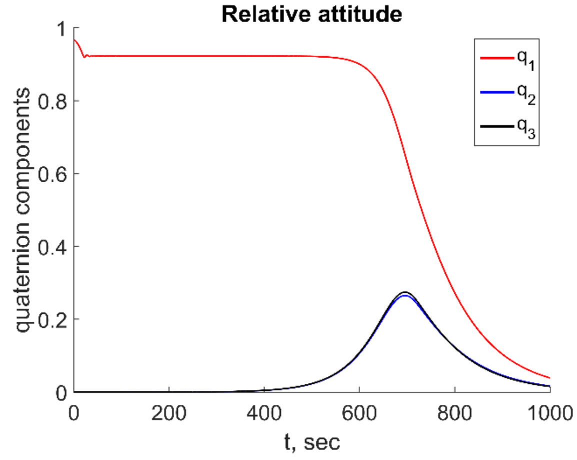
Local function

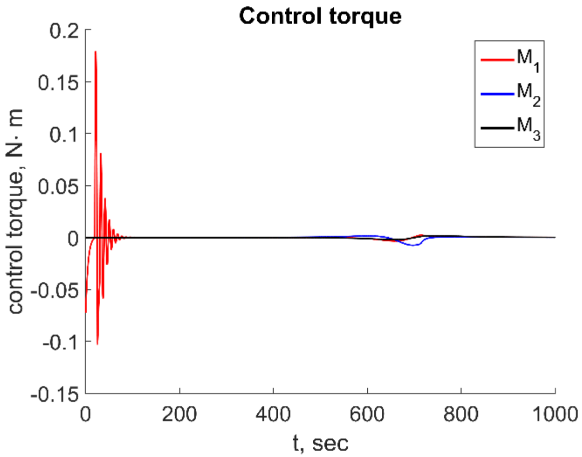


$$\mathbf{J} = \mathrm{diag}(20,\ 30,\ 40)$$

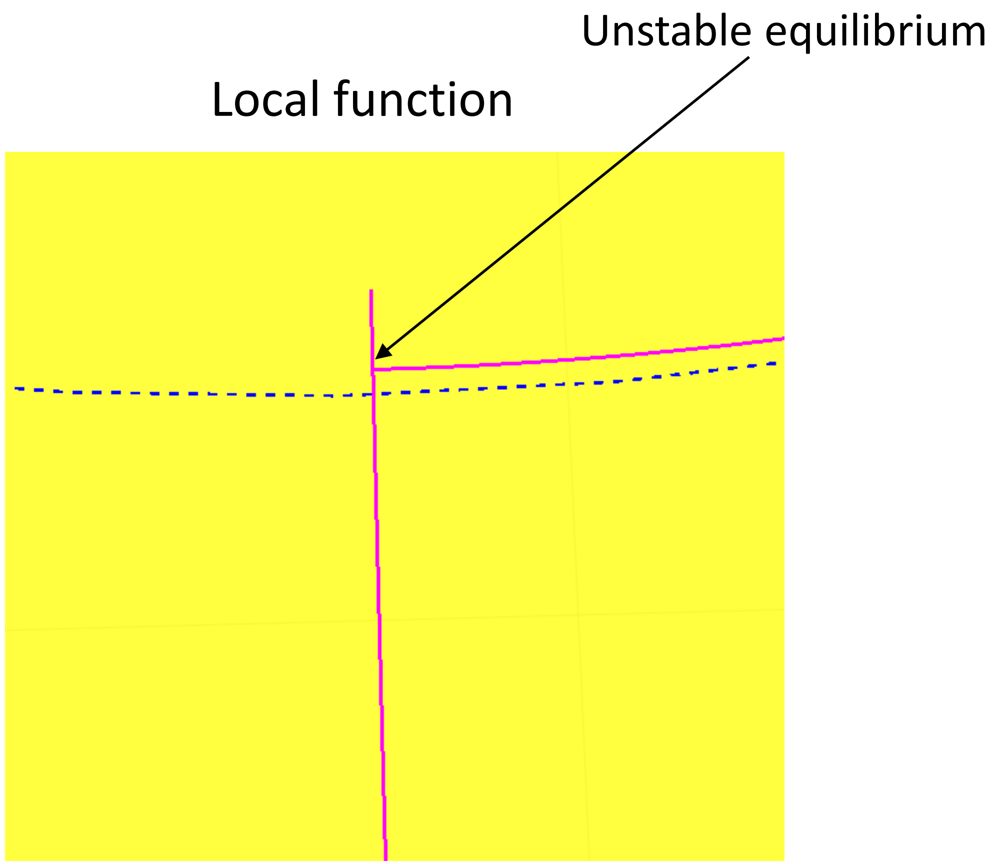
$$\alpha_i = \frac{\pi}{6},\quad \beta_i = \frac{\pi}{4}$$

Simulation results





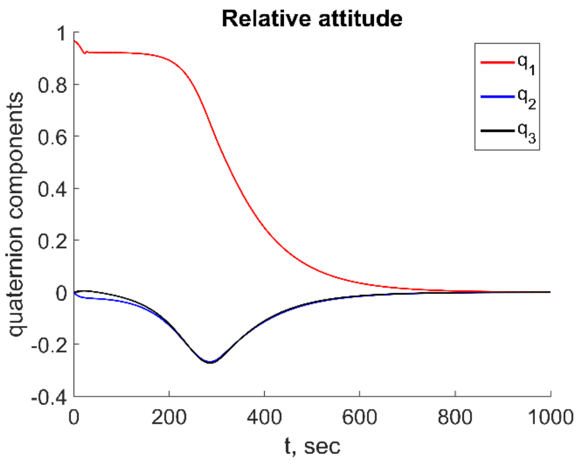
Local function

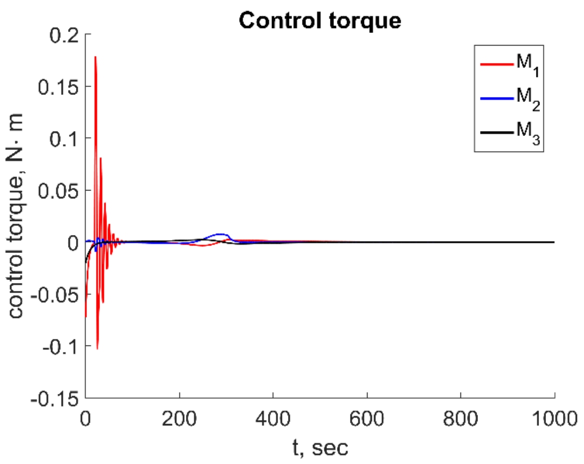


$$\mathbf{J} = \mathrm{diag}(20,\ 30,\ 40)$$

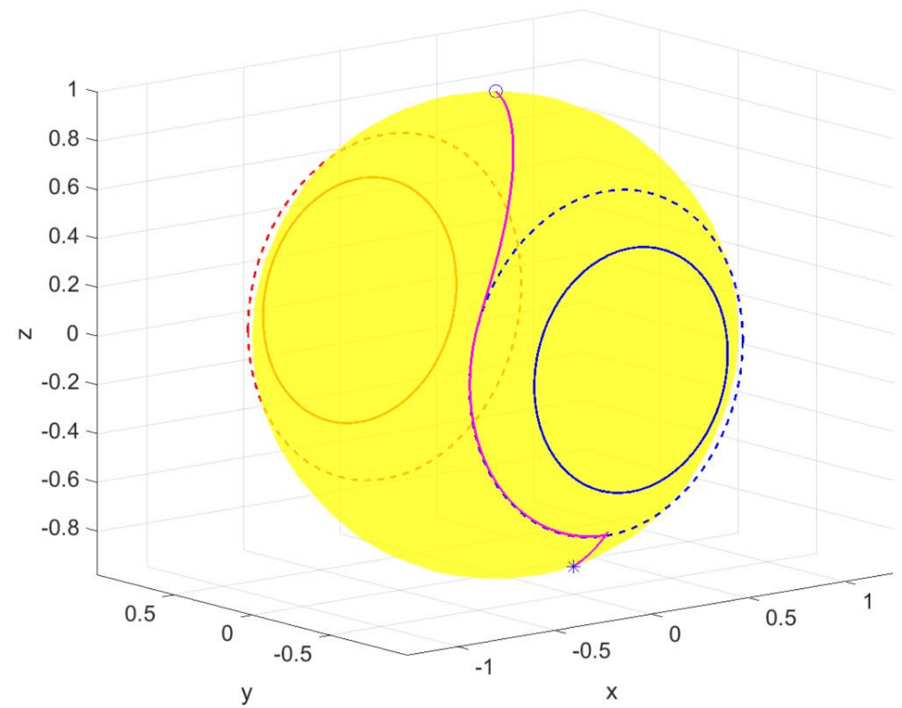
$$\alpha_i = \frac{\pi}{6}, \quad \beta_i = \frac{\pi}{4}$$

Simulation results





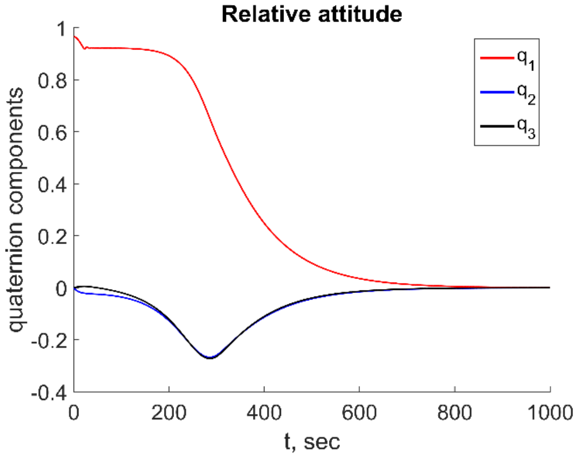
Local function

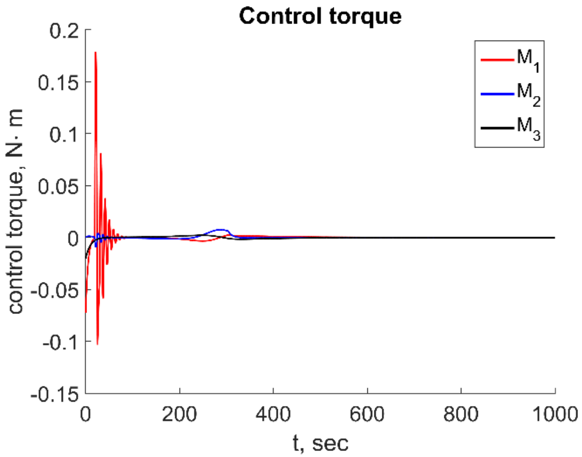


$$\mathbf{J} = \mathrm{diag}(20,\ 30,\ 40) \qquad \alpha_i = \frac{\pi}{6},\quad \beta_i = \frac{\pi}{4}$$

$$\boldsymbol{\omega}_0 = \begin{pmatrix} 0 & 0 & 5\cdot 10^{-3} \end{pmatrix}$$

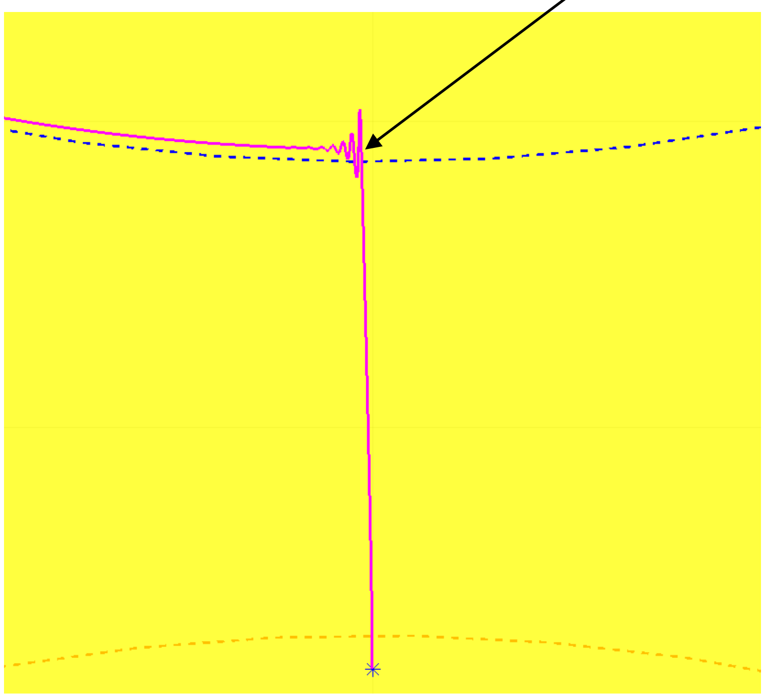
Simulation results





Unstable equilibrium

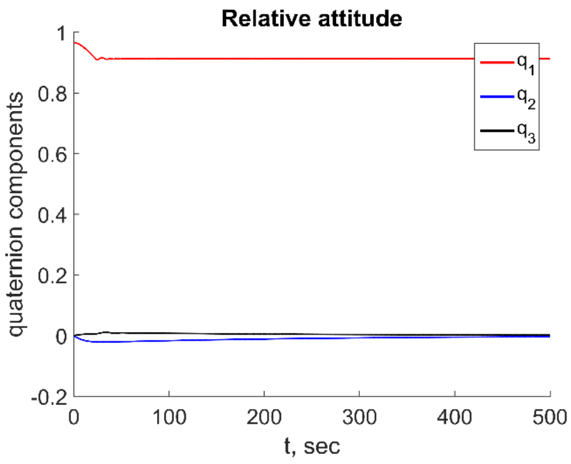
Local function

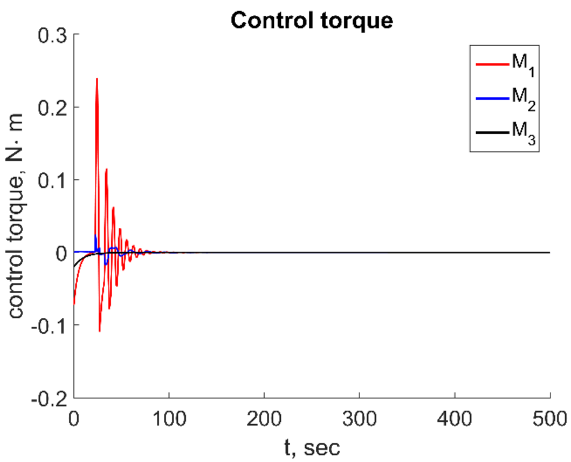


$$\mathbf{J} = \text{diag}(20,\ 30,\ 40) \qquad \alpha_i = \frac{\pi}{6},\quad \beta_i = \frac{\pi}{4}$$

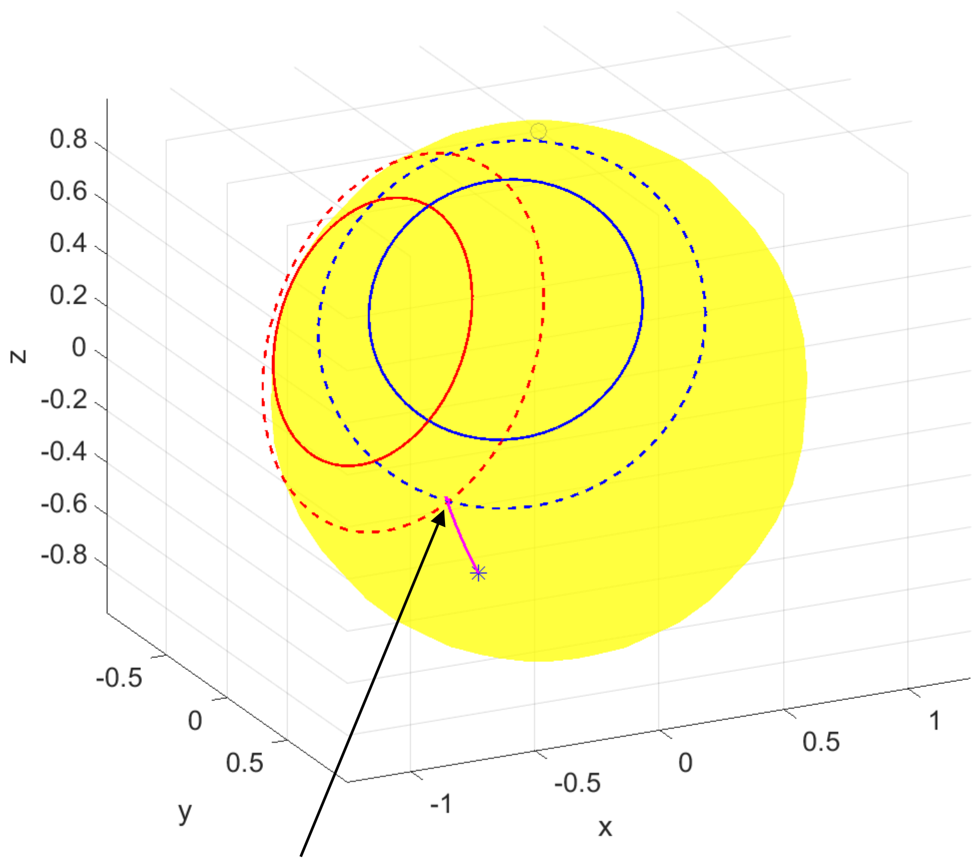
$$\boldsymbol{\omega}_0 = \begin{pmatrix} 0 & 0 & 5\cdot 10^{-3} \end{pmatrix}$$

Two intersected cones



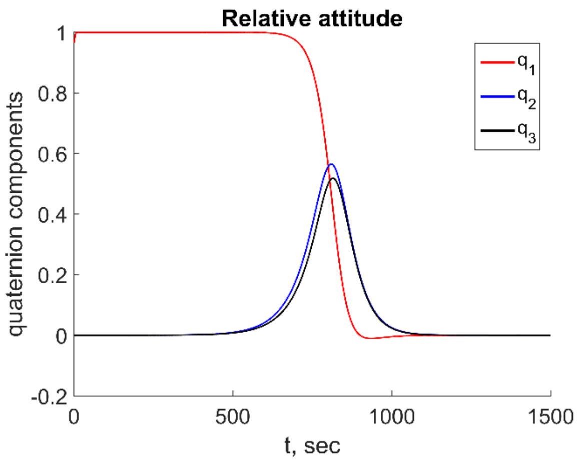


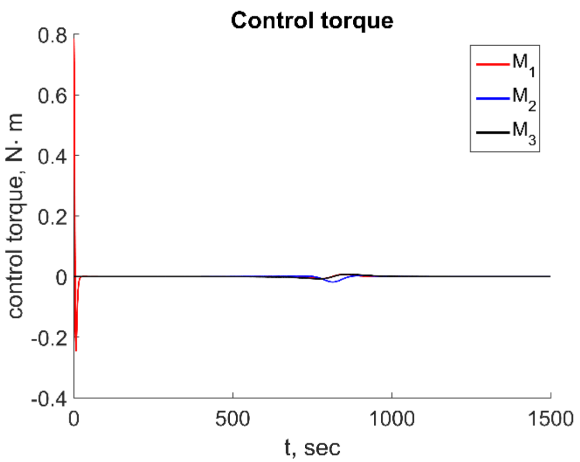
Local function



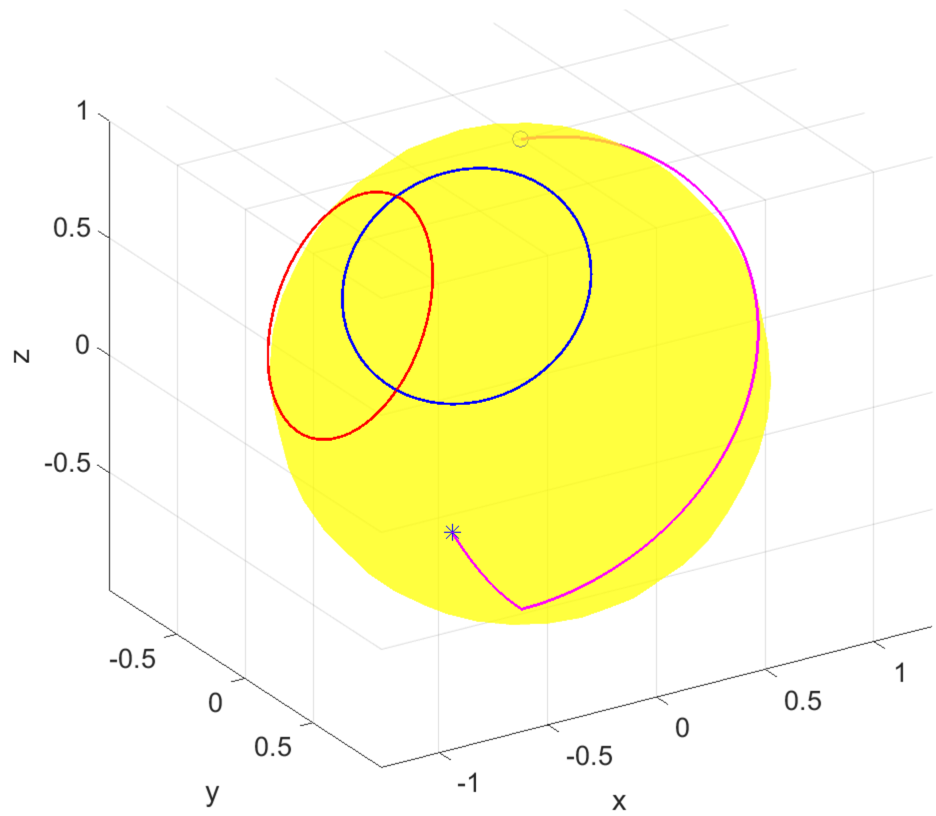
Asymptotically stable equilibrium

Two intersected cones





Global function



Conclusion

- The Lyapunov based attitude control is used for the problem of slew maneuver realization
- Two different modifications of the Lyapunov approach are suggested
- There are some problems: unstable (in case of two intersected conses – asymptotically stable) equilibrium, long convergence time
- Further investigation of the relation between the control constants and convergence time is necessary

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