Gain Selection for Attitude Stabilization of Earth-pointing Spacecraft using Magnetorquers

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Outline

- magnetorquers
- Earth-pointing stabilization
- spacecraft and geomagnetic field models
- attitude controller and gain selection
- case study

Magnetorquers

- coils placed on the spacecraft along three orthogonal axes

- interaction of magnetic dipole moment generated by coils and geomagnetic field creates torque that aligns dipole moment with field

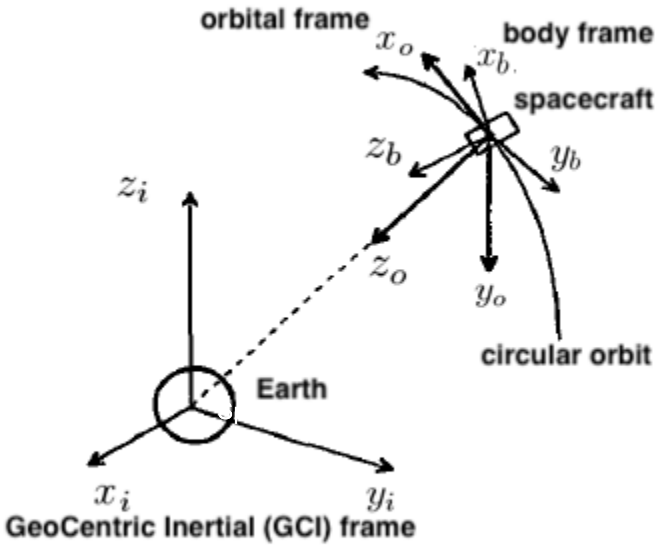
- pros
  - simpler and more reliable than other torque actuators
  - need only electrical power (no propellant)
  - generate smooth torque (no coupling with flexible modes)
  - save weight

- cons
  - cannot generate torque along geomagnetic field
  - generate small torque (slower and less accurate maneuvers)

- widely used on cubesats

Earth-pointing attitude stabilization

spacecraft with magnetorquers on a circular low Earth orbit



objective: stabilize attitude using only magnetorquers so that body frame is aligned with orbital frame (antenna or payload pointing to Earth)

parametrize attitude of body frame with respect to orbital frame using quaternion $q = [q_1\ q_2\ q_3\ q_4]^T = [q_v\ q_4]$ q_v vector part of q

body frame aligned with orbital frame $\Leftrightarrow q_v = 0\ \ q_4 = \pm 1$

Spacecraft model

attitude kinematics

$$\begin{aligned} \dot{q}_v &= \tfrac{1}{2}(q_v \times \omega^b_{bo} + q_4 \omega^b_{bo}) \\ \dot{q}_4 &= -\tfrac{1}{2} q_v^T \omega^b_{bo} \end{aligned}$$

attitude dynamics

$$J\dot{\omega}^b_{bi} = -\omega^b_{bi} \times J\omega^b_{bi} + T_{gg} + T_{coils} + T_{dist}$$

$$\omega^b_{bi} = \omega^b_{bo} + \omega^b_{oi} \qquad \omega^b_{oi} = C(q)\omega^o_{oi}$$

$C(q)$ attitude matrix

n (constant) orbital rate

$$\omega^o_{oi} = \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix}$$

attitude dynamics $J\dot{\omega}^b_{bo} = f(q, \omega^b_{bo}) + T_{coils} + T_{dist}$

m_{coils} magnetic dipole moment generated by magnetorquers

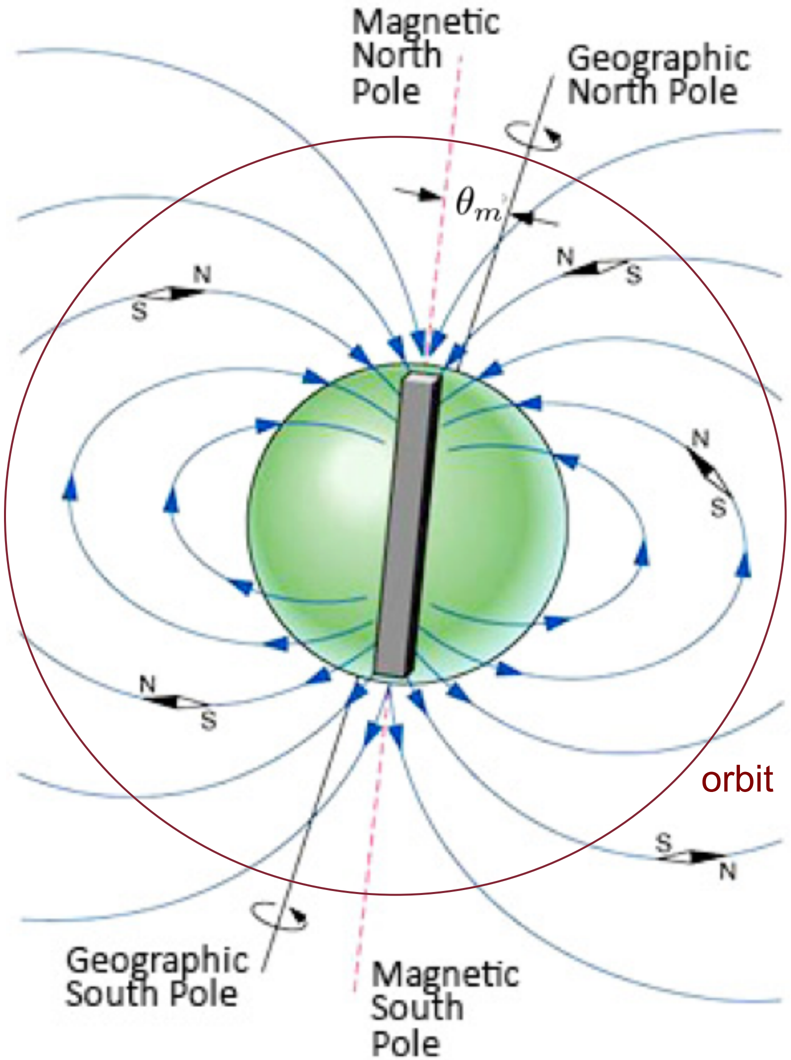
b^b geomagnetic field in body frame

$$T_{coils} = m_{coils} \times b^b = -b^b \times m_{coils} = -[C(q)b^o] \times m_{coils}$$

spacecraft model

$$\begin{aligned} \dot{q}_v &= \tfrac{1}{2}(q_v \times \omega^b_{bo} + q_4 \omega^b_{bo}) \\ \dot{q}_4 &= -\tfrac{1}{2} q_v^T \omega^b_{bo} \\ J\dot{\omega}^b_{bo} &= f(q, \omega^b_{bo}, J, n) - [C(q)b^o] \times m_{coils} + T_{dist} \end{aligned}$$

Dipole model of geomagnetic field



$\theta_m = 10.0^\circ$

$b^o(t)$ almost periodic

$\theta_m \approx 0^\circ$

$b^o(t)$ periodic with period T_{orbit}

Control law

$$m_{coils} = -b^b \times C(q)(K_p q_v + K_d \omega_{bo}^b)$$

K_p K_d 3 x 3 matrices

PD-like control with **matrix gains**

usually $K_p = k_p I_{3\times 3}$ $K_d = k_d I_{3\times 3}$

[Wisniewski and Blanke 1999] [Teofilatto et al. 2013]

matrix gains → more degrees of freedom in control action

selection of matrix gains: no trial-and-error, no exhaustive search

Matrix gain selection 1/2

$T_{dist} = 0 \qquad \theta_m = 0^\circ \;\Rightarrow\; b^o(t)$ periodic with period T_{orbit}

linearize closed-loop system

$$
\begin{aligned}
\dot{q}_v &= \tfrac{1}{2}(q_v \times \omega^b_{bo} + q_4 \omega^b_{bo}) \\
\dot{q}_4 &= -\tfrac{1}{2} q_v^T \omega^b_{bo} \\
J\dot{\omega}^b_{bo} &= f(q, \omega^b_{bo}, J, n) + \{[C(q) b^o(t)]^\times\}^2 C(q)(K_p q_v + K_d \omega^b_{bo})
\end{aligned}
$$

about $q_v = 0 \quad q_4 = 1 \quad \omega^b_{bo} = 0$

$$\Downarrow$$

$$\dot{x} = Ax + B(t)u \qquad u = -Kx$$

$$x = \begin{bmatrix} q_v \\ \omega^b_{bo} \end{bmatrix} \quad A = \begin{bmatrix} 0_{3\times3} & \tfrac{1}{2} I_{3\times3} \\ A_{21} & A_{22} \end{bmatrix} \quad B(t) = \begin{bmatrix} 0_{3\times3} \\ -J^{-1}\{[b^o(t)]^\times\}^2 \end{bmatrix} \quad K = [K_p \quad K_d]$$

$B(t)$ periodic with period T_{orbit}

Matrix gain selection 2/2

$\dot{x} = Ax + B(t)u \quad u = -Kx$ $\quad B(t)$ periodic with period T_{orbit}

select K that minimizes

$$E\left\{\int_0^\infty \left[x^T(t)Qx(t) + u(t)^T Ru(t)\right]\right\}$$

$E\{\cdot\}$ taken over set of possible $x(0) = x_0$

x_0 random variable, zero mean $E\{x_0 x_0^T\} = X_0$ [Viganò et al. 2010]

design parameters

Q 6 x 6 positive semidefinite matrix $\quad R$ 3 x 3 positive definite matrix

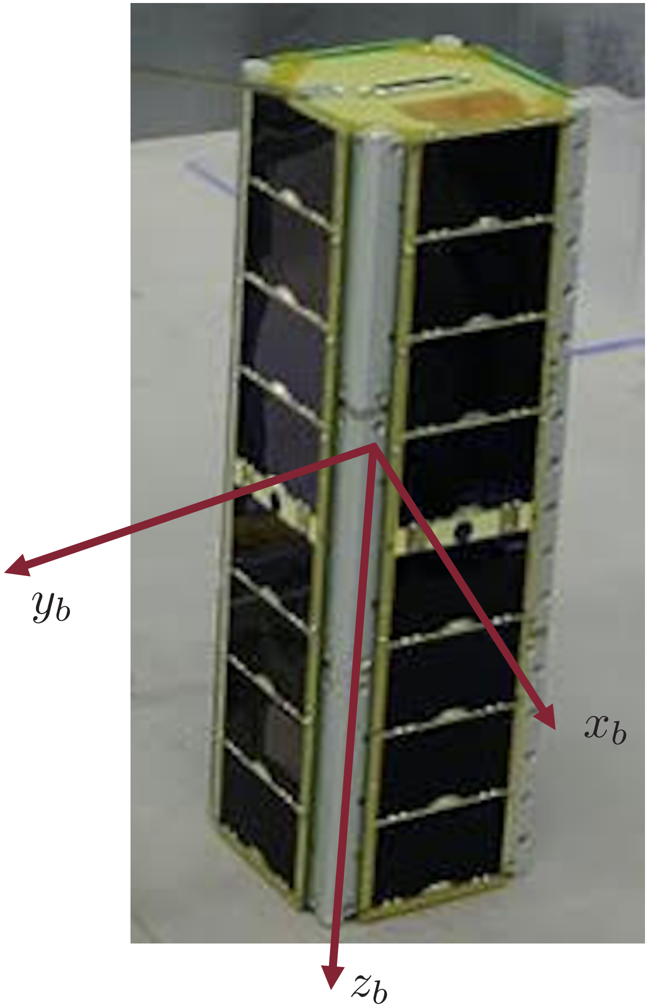
X_0 6 x 6 positive semidefinite matrix

usually $Q = \alpha I_{6\times 6} \quad R = I_{3\times 3} \quad X_0 = \sigma_0^2 I_{6\times 6}$

design parameters: $\alpha \quad \sigma_0^2$

Case study

Tigrisat



$$J_x = J_y = 4.09 \cdot 10^{-2} \text{ kg m}^2 \qquad J_z = 6.5 \cdot 10^{-3} \text{ kg m}^2$$

circular orbit altitude = 629 km $T_{orbit} = 5832$ sec

inclination = $97°$ RAAN = $68.5°$

selection of K_p and K_d

$$Q = I_{6\times6} \quad R = I_{3\times3} \quad E\{x_0 x_0^T\} = I_{6\times6}$$

initial guess $K_p = 300\ I_{3\times3} \quad K_d = 1.8 \cdot 10^4\ I_{3\times3}$

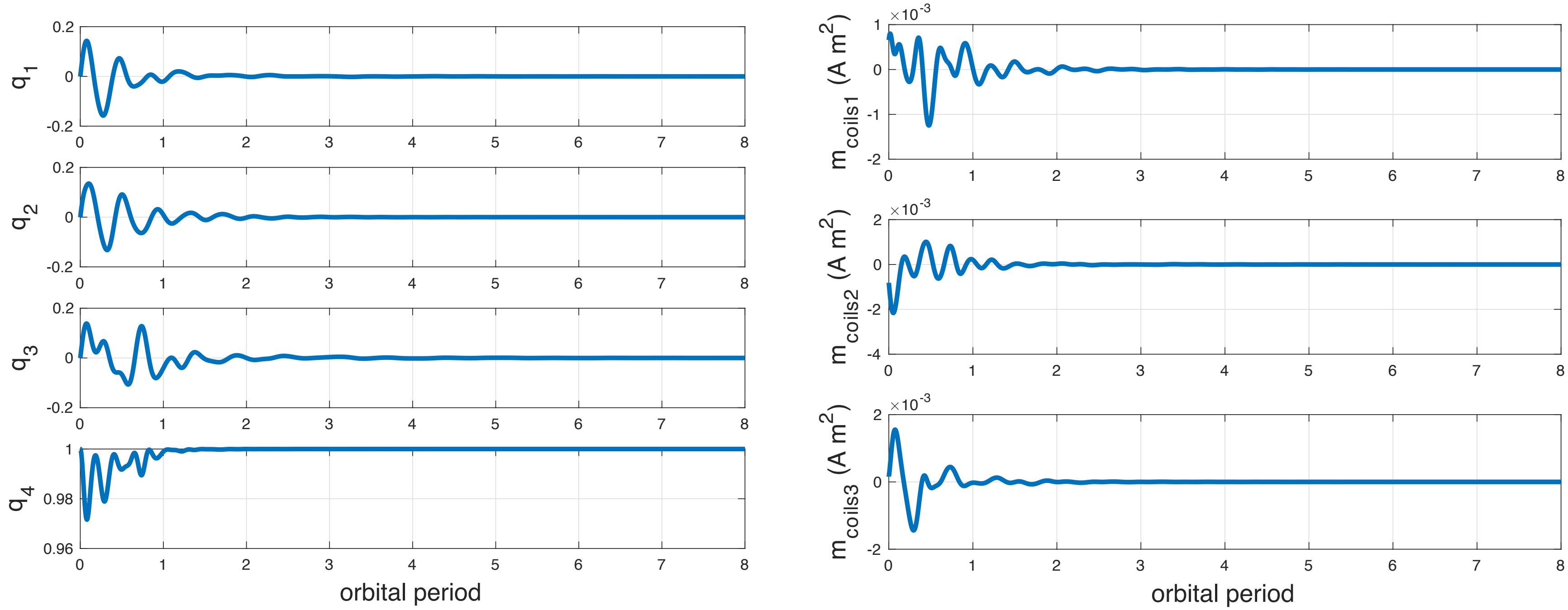
$$\Downarrow$$

$$K_p = \begin{bmatrix} 293.4863 & 0.5515 & -9.7049 \\ -0.0069 & 299.8118 & -4.1120 \\ 4.8505 & -0.1118 & 299.8613 \end{bmatrix}$$

$$K_d = \begin{bmatrix} 1.8 \cdot 10^4 & 0 & 0 \\ 0 & 1.8 \cdot 10^4 & 0 \\ 0 & 0 & 1.8 \cdot 10^4 \end{bmatrix}$$

Simulation

initial state $q_v(0) = 0 \quad q_4(0) = 1 \quad \omega^b_{bo}(0) = [10^{-3}\ 10^{-3}\ 10^{-3}]^T \quad \text{rad/s}$

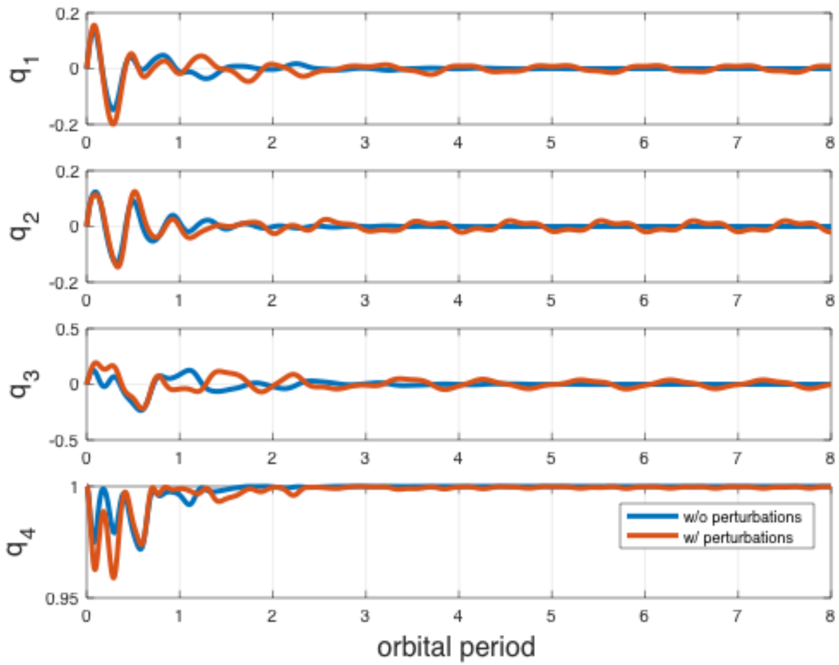


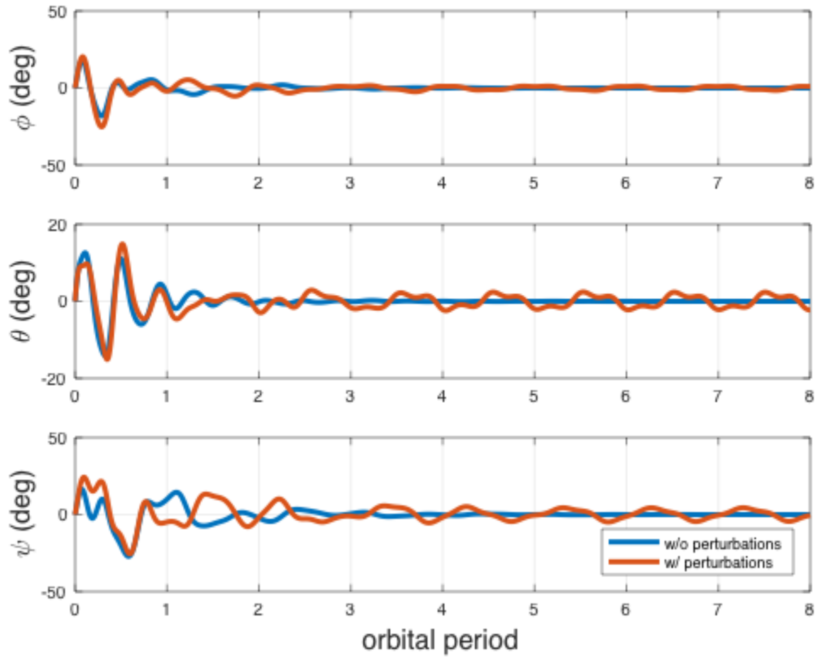
Simulation

initial state $q_v(0) = 0 \quad q_4(0) = 1 \quad \omega_{bo}^b(0) = [10^{-3}\ 10^{-3}\ 10^{-3}]^T \quad \mathrm{rad/s}$

perturbations: $\theta_m = 10^\circ$ + residual magnetization torque

$$m_0 = [2 \cdot 10^{-4}\ \ 2 \cdot 10^{-4}\ \ 2 \cdot 10^{-4}]^T\ \mathrm{A\ m}^2$$





Conclusion

- attitude stabilization of Earth-pointing spacecraft using magnetorquers

- PD-like control law with matrix gains

- selection of matrix gains based on periodic linear quadratic problem