4th IAA Conference on University Satellite Missions and CubeSat Workshop

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Plasma Brake Approximate Trajectory. Part II: Relative Motion

Outline

1. Introduction
   - Contents of the work
   - Plasma brake concept
2. Mathematical model
   - Plasma brake acceleration model
   - Linearized equations of relative motion
   - Orbital decay
   - Relative trajectory
   - Iterative process
3. Mission application
   - Cases studied
4. Conclusions

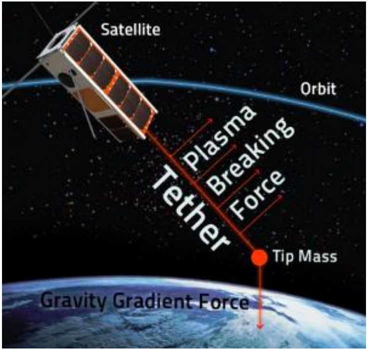
Introduction

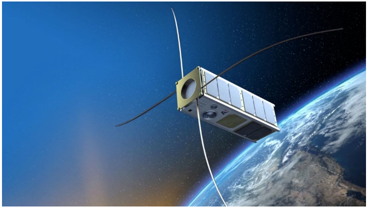
Contents of the work

- Investigation of **plasma brake** option for space **debris mitigation**
- Analytical solutions to the **linearized** Hill-Clohessy-Wiltshire equations to evaluate the **decay** of a plasma brake-based **nanosatellite** from low Earth orbit
- Estimation of the **decay time** as a function of the plasma brake acceleration at a reference altitude
- Results **consistent** with those obtained in the companion paper (Niccolai et al., 2017)

Plasma brake concept

- **Propellantless** de-orbiting system
- An **electrically charged tether** interacts with the **ionized particles** of the upper layers of the Earth's atmosphere
- The tether is **gravity-gradient** stabilized
- The Coulomb drag is proportional to the **tether length** and is treated as a function of the **orbital radius** only (according to Orsini et al., 2017)
- Ongoing **ESTCube-2** and **Aalto-1** missions will attempt the use of a plasma brake to de-orbit the satellite after end-of-life





Mathematical model

Plasma brake acceleration (Orsini et al., 2017)

Plasma brake acceleration $\boldsymbol{a}_T$ as a function of the **orbital radius** r

$$\boldsymbol{a}_T = -a_{T_0}\, \exp\left[-\frac{m_\mathrm{i}\, g_0\, R_\oplus^2}{4\, k_B\, T}\, \left(\frac{r - R_\oplus}{r^2} - \frac{r_0 - R_\oplus}{r_0^2}\right)\right] \hat{\boldsymbol{v}}$$

Definitions

- a_{T_0} = plasma brake-induced acceleration at the **reference** orbital radius r_0
- m_i = mean **molecular mass** of the incoming flow
- g_0 = standard **gravity**
- $R_\oplus$ = Earth's mean radius
- k_B = Boltzmann constant
- T = constant ionosphere **temperature**
- r = orbital radius
- $\hat{\boldsymbol{v}}$ = orbital velocity **unit vector**

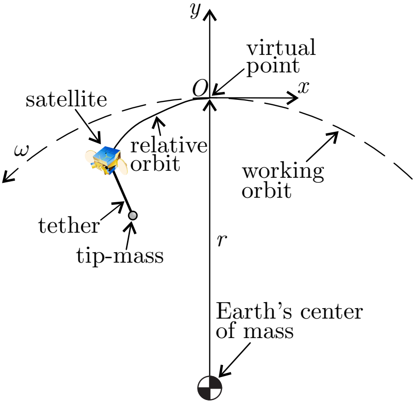
Plasma brake relative dynamics

Linearized Hill-Clohessy-Wiltshire equations with $\omega \triangleq \sqrt{\mu_\oplus / r^3}$ and a_T (**constant** value)

$$\begin{cases} \ddot{x} - 2\,\omega\,\dot{y} = a_T \\ \ddot{y} + 2\,\omega\,\dot{x} - 3\,\omega^2\,y = 0 \end{cases}$$

with $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0$

$$\begin{cases} x(t) = -\dfrac{3\,a_T\,t^2}{2} + \dfrac{4\,a_T}{\omega^2}\left[1 - \cos(\omega t)\right] \\ y(t) = -\dfrac{2\,a_T\,t}{\omega}\left[1 - \dfrac{\sin(\omega t)}{\omega t}\right] \end{cases}$$



Note

- The solutions hold until $\sqrt{x^2 + y^2}/r \ll 1$
- Since $a_T \ll g_0$, the satellite osculating orbit is nearly circular

Orbital decay evaluation after N revolutions

After an integer number N of **revolutions** of the satellite **virtual point**

$$t_N \triangleq \frac{2\pi}{\omega} N \quad \Rightarrow \quad \begin{cases} x(t_N) = -\dfrac{6\pi^2\, a_T\, N^2}{\omega^2} \\ y(t_N) = -\dfrac{4\pi\, a_T\, N}{\omega^2} \end{cases}$$

The **change** in orbital radius is

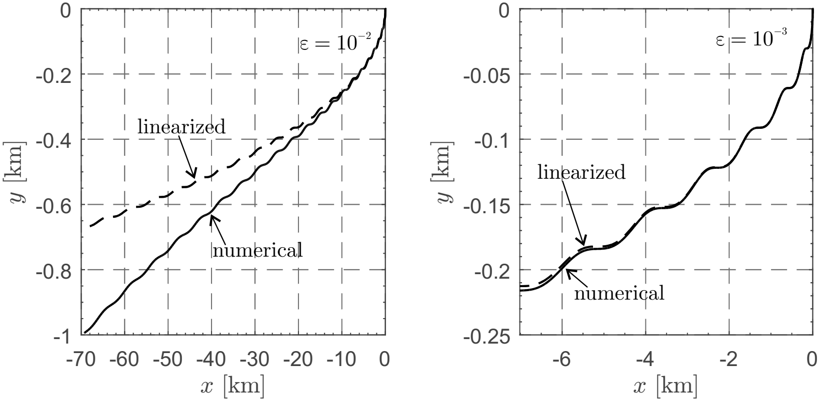
$$\Delta r = r \left[1 - \sqrt{1 + \frac{8\pi\, a_T\, N\, r^2}{\mu_\oplus} \left(\frac{9\pi^3\, a_T\, N^3\, r^2}{2\,\mu_\oplus} + \frac{2\pi\, a_T\, N\, r^2}{\mu_\oplus} - 1 \right)} \right]$$

where $\mu_\oplus$ is the Earth's **gravitational parameter**, while

$$N = \frac{\sqrt{2}}{3\pi} \sqrt{\sqrt{1 + \frac{9\,\epsilon^2\,\mu_\oplus^2}{16\, a_T^2\, r^4}} - 1} \quad \text{with} \quad \epsilon \triangleq \max_r \left\{ \sqrt{x(t_N)^2 + y(t_N)^2} / r \right\}$$

Relative trajectory

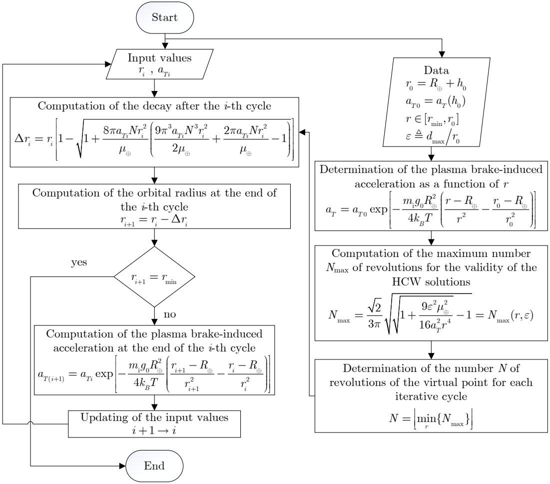
Comparison between the **approximate** relative trajectory and the results obtained by **numerical integration** ($a_{T0} = 0.0024$ mm/s^2 and $h_0 \triangleq r_0 - R_\oplus = 1000$ km)



The **linearized solutions** are **acceptable** if $\underline{\epsilon \leq 10^{-3}}$

Iterative process for the decay analysis

The Hill's linearized solutions are **implemented** in an **iterative process** to compute the **altitude loss** and the **decay time**



- N is **computed** once and for all at the **initialization** of the iteration
- r is **updated** at the end of each cycle (**step** function)
- **Decay** time:

$$t_d = N\sum_i \frac{2\pi}{\omega_i}$$

Mission application

Mission application

Characteristics of the satellites in the plasma brake-enabled de-orbiting simulations, initial altitude $h_0 = 1000\,$km

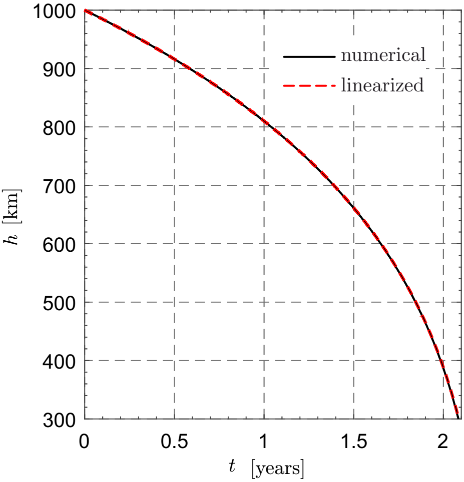
	m [kg]	L_t [m]	$\lvert V_t \rvert$ [V]	a_{T_0} [mm/s^2]
spacecraft ①	1	25	500	0.0014
spacecraft ②	4	100	1000	0.0020
spacecraft ③	**10**	**300**	**1000**	**0.0024**

Decay time t_d and **computational** time t_c for the three cases studied, obtained through **numerical integration** (reference solution) and **algorithm** (approximate solution)

	Numerical integration		Approximate method		
	t_d [years]	t_c [s]	t_d [years]	t_c [s]	error [%]
spacecraft ①	3.5632	2061	3.5697	0.1	0.1835
spacecraft ②	2.5006	1356	2.5026	0.1	0.0794
spacecraft ③	**2.0838**	**1162**	**2.0859**	**0.1**	**0.0969**

Example: case of spacecraft ③

- Comparison between the **altitude loss** evaluated through the **algorithm** (dashed line) and that obtained via **numerical integration** (solid line) of the non-linearized equations of motion
- $a_{T_0} = 0.0024\,\mathrm{mm/s^2}$
- **Few years** to de-orbit a plasma brake-based satellite from $1000\,\mathrm{km}$ of altitude if $\underline{a_{T0} \simeq 10^{-3}\,\mathrm{mm/s^2}}$



Note: Conservative evaluation of the decay time (atmospheric drag not included in the model)

Conclusions

Conclusions

- The **analytical** solutions to the **linearized** Hill's equations have been **implemented** in an **iterative process** to compute the decay time of a **plasma brake**-based nanosatellite
- The plasma brake **acceleration** is a **function** of the **orbital radius** only
- The results show **negligible errors** in terms of decay time when compared to the outputs of an orbital propagator (of the order of 0.1%)
- The **computational time** is **reduced** by **four orders of magnitude** with respect to numerical integration
- The **decay times** of a plasma brake-based satellite are in accordance with the Inter-Agency Space Debris Coordination Committee (**IADC**) guidelines

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