The 4th IAA Conference on University Satellite Missions & CubeSat Workshop

Rome, Italy, 4 – 7 December 2017

Lorenzo Niccolai, **Marco Bassetto**,
Alessandro A. Quarta, & Giovanni Mengali

Department of Civil and Industrial Engineering

Università di Pisa

Plasma Brake Approximate Trajectory
Part I: Geocentric Motion

Outline

1. Introduction
   - Aim of the work
   - Plasma brake concept
2. Mathematical model
   - Equations of motion
   - Variational equations of modified parameters
   - Series expansion
   - Evolution of modified parameters
3. Approximate plasma brake model
   - Coulomb drag expression
   - Coulomb drag analytical approximation
4. Rectification procedure
5. Case study
6. Conclusions

Introduction

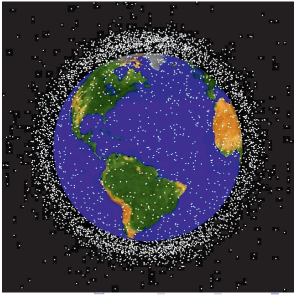
Aim of the work

Motivation

- Space debris is a major concern, especially for LEO missions.
- IADC guidelines: residual orbit life ≤ 25 years for out-of-order satellites.
- Plasma brake is a promising innovation in terms of deorbiting technologies.

Aim of the work

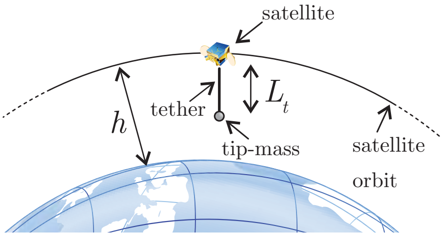
- Analytical approximation of the **geocentric trajectory** of a spacecraft equipped with a plasma brake
- Estimation of the **decay time**



Plasma brake concept

Plasma brake

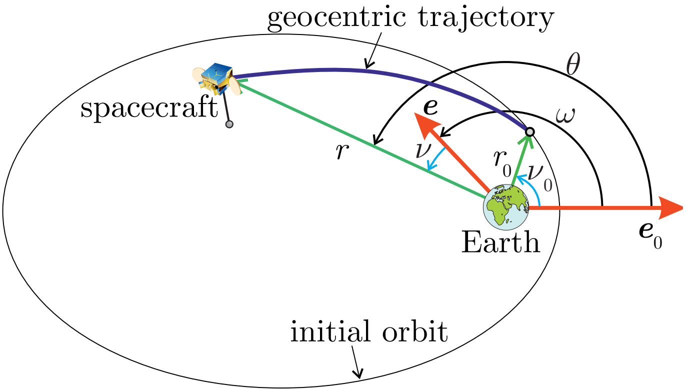
- Derivation of the **electric solar wind sail** (E-sail) concept
- Application: **deorbiting** from LEO



Working principle

- A tether is released in the **ionosphere** and **electrostatically charged** by a voltage source.
- The collisions between ions and tether generate a **Coulomb drag**.
- The Coulomb drag **lowers the spacecraft altitude**.

Geometrical sketch of the problem



Nomenclature

- $r \triangleq$ geocentric distance
- $\theta \triangleq$ angular coordinate
- $\nu \triangleq$ true anomaly
- $\omega \triangleq$ apse line rotation
- $\boldsymbol{e} \triangleq$ eccentricity vector

Note: subscript 0 identifies initial conditions.

Mathematical model

2D Equations of motion

- Polar reference frame
- Two-dimensional motion (thrust vector belongs to the orbital plane)
- Coulomb drag opposite to spacecraft velocity

$$\begin{cases} \ddot{r} - r\dot{\theta}^2 = -\dfrac{\mu_\oplus}{r^2} - \epsilon\,\dfrac{\mu_\oplus}{r_0^2}\,\dfrac{\dot{r}}{\sqrt{\dot{r}^2 + (r\,\dot{\theta})^2}} \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\epsilon\,\dfrac{\mu_\oplus}{r_0^2}\,\dfrac{r\,\dot{\theta}}{\sqrt{\dot{r}^2 + (r\,\dot{\theta})^2}} \end{cases}$$

Nomenclature

- $\mu_\oplus \triangleq$ Earth's gravitational parameter
- $D_c \triangleq$ Coulomb drag magnitude
- $\epsilon \triangleq D_c/\sqrt{\mu_\oplus/r_0^2}$

Problems

- Coulomb drag is very complex to model.
- The integration has a high computational cost.

Non-singular orbital elements

Introduction of **non-dimensional orbital parameters** (Bombardelli et al., 2011):

$$q_1 \triangleq \frac{e}{\widetilde{H}} \cos\omega, \qquad q_2 \triangleq \frac{e}{\widetilde{H}} \sin\omega, \qquad q_3 \triangleq \frac{1}{\widetilde{H}}$$

where $\widetilde{H}$ is the dimensionless angular momentum magnitude

Note: classical orbital parameters can be recovered as

$$a = \frac{r_0}{q_3^2 - q_1^2 - q_2^2} \qquad \omega = \arctan\left(\frac{q_2}{q_1}\right)$$

$$e = \frac{\sqrt{q_1^2 + q_2^2}}{q_3} \qquad v_r = \sqrt{\frac{\mu_\odot}{r_0}}\,(q_1\,\sin\theta - q_2\,\cos\theta)$$

$$r = \frac{r_0}{q_3^2 + q_1\,q_3\cos\theta + q_2\,q_3\sin\theta} \qquad v_\theta = \sqrt{\frac{\mu_\odot}{r_0}}\,(q_1\,\cos\theta + q_2\,\sin\theta + q_3)$$

Variational equations of modified parameters

The **variational equations of modified parameters** are (Coulomb drag opposite w.r.t. orbital velocity)

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{\epsilon}{q_3\, s^3 \sqrt{e^2 + 2\, e\, \cos\nu + 1}} \begin{bmatrix} s\, \sin\theta & (s+q_3)\, \cos\theta \\ -s\, \cos\theta & (s+q_3)\, \sin\theta \\ 0 & -q_3 \end{bmatrix} \begin{bmatrix} e\, \sin\nu \\ 1 + e\, \cos\nu \end{bmatrix}$$

where

$$s = q_1 \cos\theta + q_2 \sin\theta + q_3, \qquad e = \frac{\sqrt{q_1^2 + q_2^2}}{q_3}, \qquad \nu = \theta - \arctan\left(\frac{q_2}{q_1}\right)$$

Initial conditions:

$$q_1 = \frac{e_0}{\widetilde{H}_0}, \qquad q_2 = 0, \qquad q_3 = \frac{1}{\widetilde{H}_0} \qquad \text{with} \quad \widetilde{H}_0 = \sqrt{1 + e_0 \cos\nu_0}$$

Introduction to expansion method

Fundamental hypothesis

The Coulomb drag magnitude is small w.r.t. the Earth's gravity, i.e. $\epsilon \ll 1$

The dimensionless propulsive acceleration ϵ is a **perturbation term** in the **asymptotic series expansion**:

$$q_1 = q_{10} + \epsilon\, q_{11} + O(\epsilon^2)$$

$$q_2 = q_{20} + \epsilon\, q_{21} + O(\epsilon^2)$$

$$q_3 = q_{30} + \epsilon\, q_{31} + O(\epsilon^2)$$

where

- q_{i0} are (constant) unperturbed terms
- q_{i1} are **first order** perturbation terms

Evolution of modified parameters

Mathematical procedure

1. The series expansions are substituted in the variational equations.
2. Terms of the same perturbative order are equated.
3. Differential equations are integrated in the angular variable, maintaining constant the value of the perturbative parameter ϵ.

Analytical expressions of modified parameters (**circular initial orbit**)

$$q_1 = -2\,\epsilon\,(\sin\theta - \sin\theta_0)$$

$$q_2 = 2\,\epsilon\,(\cos\theta - \cos\theta_0)$$

$$q_3 = 1/\widetilde{H}_0 + \epsilon\,(\theta - \theta_0)$$

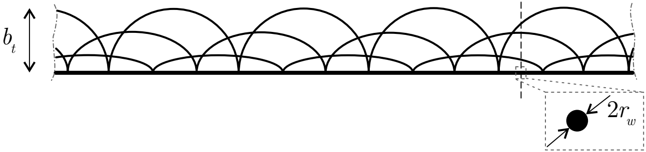
More complex expressions exist for the **elliptic initial orbit** case.

Approximate plasma brake model

Coulomb drag

Hypothesis

- Earth's magnetosphere effects are neglected (Janhunen, 2014).
- A constant drag per unit length is assumed.
- A **Heytether** is considered.



Nomenclature

- $el \triangleq$ elementary charge
- $m_i \triangleq$ ions mass
- $n \triangleq$ plasma bulk density
- $\epsilon_0 \triangleq$ vacuum permittivity

Coulomb drag from a tether with length L_t and (negative) electric potential V_t (Janhunen, 2014)

$$D_c = 3.864\, L_t\, m_i\, n\, v^2 \sqrt{\frac{\epsilon_0\, V_a}{el\, n}} \exp\left(-\frac{m_i\, v^2}{2\, el\, V_a}\right) \quad \text{with} \quad V_a \triangleq \frac{2\,|V_t|}{\ln\left(\dfrac{\epsilon_0\,|V_t|}{el\, n_0\, b_t\, r_w}\right)}$$

Approximate model (Orsini et al., 2017)

- Plasma density estimated with **geopotential model**
- Constant ions mass (atomic oxygen, $m_i \simeq 16\,\mathrm{u}$) and temperature
- Initial altitude taken as **reference altitude**

$$D_c \simeq D_{c0} \sqrt{\frac{n}{n_0}} \left(\frac{v}{v_0}\right)^2 f_1(v, v_0)\, f_2(n, n_0, v, v_0)\, f_3(n, n_0)$$

D_{c0} is calculated with accurate ionosphere models (IRI).

Approximation for LEOs ($300\,\mathrm{km} \leq h \leq 1000\,\mathrm{km}$) with $\widetilde{h} = 1000\,\mathrm{km}$

- $0.96 \leq f_1 \leq 1 \qquad 1 \leq f_2 \leq 1.04 \qquad \Rightarrow f_1\, f_2 \simeq 1$
- $(v/v_0)^2 \simeq 1$ **Conservative assumption** with max error 10%
- $f_3 \simeq 1$ **Conservative assumption** with max error 16.5%

$$D_c \simeq D_{c0}\, \exp\left\{-\frac{m_i\, \mu_\oplus}{4\, k_B\, T}\left[\frac{h}{(R_\oplus + h)^2} - \frac{h_0}{(R_\oplus + h_0)^2}\right]\right\}$$

Rectification procedure

Rectification procedure (1/2)

1. Select the **rectification coordinate** θ_r and calculate the corresponding values of the modified parameters q_{ir}.
2. Find the **altitude** h_r, the **eccentricity** e_r, the **true anomaly** ν_r, the **apse line rotation angle** ω_r, and the **dimensionless angular momentum** $\widetilde{H}_r$ as functions of q_{ir}.
3. **Update** the **magnitude of the Coulomb drag** at the altitude h_r by means of the approximate analytical model and calculate the new value of the perturbative parameter $\bar{\epsilon}$.
4. Define a new set of **auxiliary variables**

$$\bar{\theta} \triangleq \theta - \omega, \quad \bar{\omega} \triangleq \omega - \omega_r, \quad \bar{q}_1 \triangleq \frac{e}{\widetilde{H}} \cos\omega_r \quad \bar{q}_2 \triangleq \frac{e}{\widetilde{H}} \sin\omega_r \quad \bar{q}_3 \triangleq \frac{1}{\widetilde{H}}$$

Rectification procedure (2/2)

5. The **initial conditions** for the new auxiliary variables are

$$\bar{\theta}_0 = \nu_r, \qquad \bar{\omega}_0 = 0, \qquad \bar{q}_{10} = \frac{e_r}{\widetilde{H}_r}, \qquad \bar{q}_{20} = 0, \qquad \bar{q}_{30} = \frac{1}{\widetilde{H}_r}$$

6. Calculate the **evolution of the new auxiliary variables** with the analytical expressions, substituting e_0, $\widetilde{H}_0$, ν_0, and ϵ with e_r, $\widetilde{H}_r$, ν_r, $\bar{\epsilon}$, and $\bar{\theta}$, respectively.

7. To get the **initial parameters**, apply the following **rotational matrix**

$$\begin{bmatrix} q_1(\theta) \\ q_2(\theta) \\ q_3(\theta) \end{bmatrix} = \begin{bmatrix} \cos\omega_r & -\sin\omega_r & 0 \\ \sin\omega_r & \cos\omega_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{q}_1(\bar{\theta}) \\ \bar{q}_2(\bar{\theta}) \\ \bar{q}_3(\bar{\theta}) \end{bmatrix} \qquad \text{for} \quad \theta \geq \theta_r$$

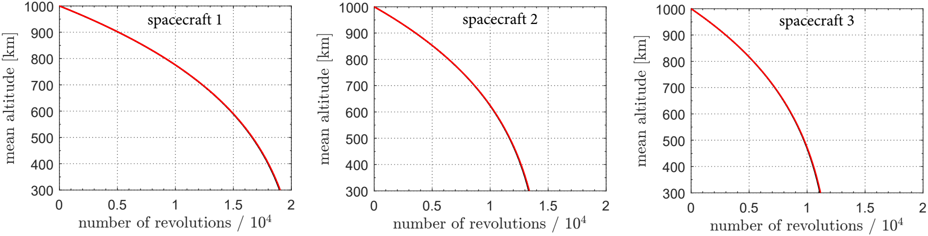
Case study

Deorbiting profile from $h_0 = 1000\,\text{km}$, mean solar activity

Three different **nanosatellites** are analyzed in our simulations

	m [kg]	L_t [m]	$\lvert V_t \rvert$ [V]
spacecraft 1	1.0	25	500
spacecraft 2	4.0	100	1000
spacecraft 3	10.0	300	1000

Comparison between the **approximate method** with **100 rectifications per year** (**red line**) and an **orbital propagator** (**black line**)



Decay times from $h_0 = 1000\,\text{km}$ to $300\,\text{km}$

	Numerical	Analytical	Percentage error
spacecraft 1	$1317\,\text{days}$	$1320\,\text{days}$	0.26%
spacecraft 2	$924\,\text{days}$	$928\,\text{days}$	0.38%
spacecraft 3	$770\,\text{days}$	$774\,\text{days}$	0.45%

Remarks

- The estimation of the **geocentric trajectory** is **practically coincident** with the output of an orbital propagator.
- The determination of the **decay time** is also **very accurate**.
- The approximated method requires a **computational cost** which is about **2 order of magnitudes smaller** w.r.t. the numerical integration of the equations of motion.
- The estimated decay times are **in accordance** with **IADC guidelines**.

Conclusions

Conclusions

- **Approximate method** for the **geocentric trajectory** analysis of a spacecraft deorbiting from a LEO by means of a **plasma brake**
- Model based on the **small magnitude** of the Coulomb drag, which is expressed as a **function of the orbital altitude only**
- Results show very **small errors**, both in terms of geocentric trajectory and decay times, when compared to the outputs of an orbital propagator
- **Computational time** about **two order of magnitudes smaller** with respect to the numerical integration of the equations of motion, even with a high number of rectifications
- Decay times of a **plasma brake-enabled deorbiting** are **in accordance with international guidelines**

4th IAA Conference on University Satellite Missions and CubeSat Workshop

Rome, Italy, 4 – 7 December 2017

Lorenzo Niccolai, **Marco Bassetto**,
Alessandro A. Quarta & **Giovanni Mengali**

Department of Civil and Industrial Engineering

IN SUPREMÆ DIGNITATIS 1343

UNIVERSITÀ DI PISA

Plasma Brake Approximate Trajectory. Part I: Geocentric Motion