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AN ANALYTICAL APPROACH TO AUTONOMOUS OPTICAL NAVIGATION FOR A CUBESAT MISSION TO A BINARY ASTEROID SYSTEM

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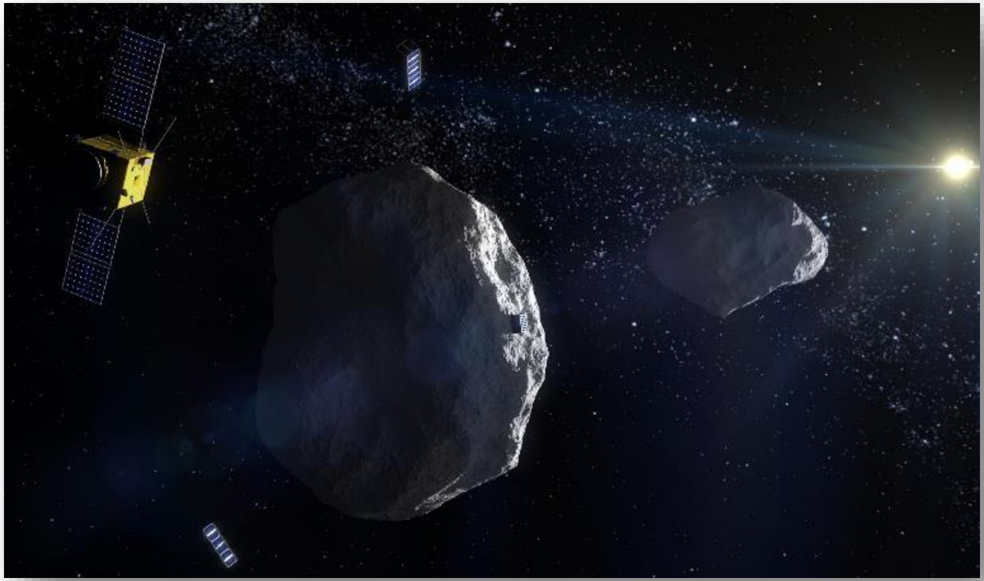
Study motivation

Interplanetary CubeSat missions are gathering a growing interest among Space Agencies.

- NASA JPL Mars Cube One (MarCO)
- ESA AIM CubeSat Opportunity Payloads (COPINS)

For such missions it is highly desirable the spacecraft orbit determination to be performed in *real time* and *autonomously*

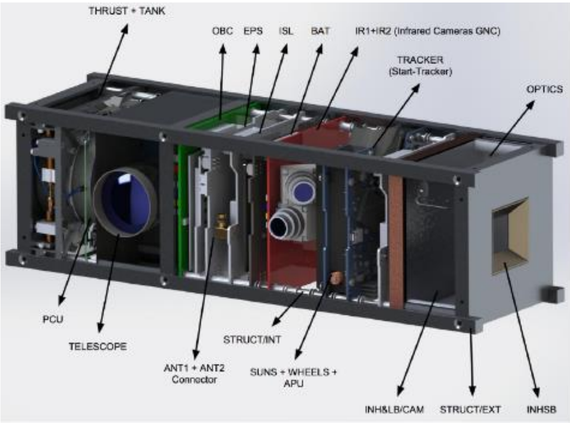


The prologue: DustCube

Mission concept by UniBo, UVigo and Micos, in response to AIM COPINS opportunity.

Mission Objectives:

- Characterization of **interplanetary particles** by in-situ and remote measurements using light scattering Nephelometers
- Characterization of **dust plume ejecta** after DART's impact on Didymoon (mass, composition, size, shape)

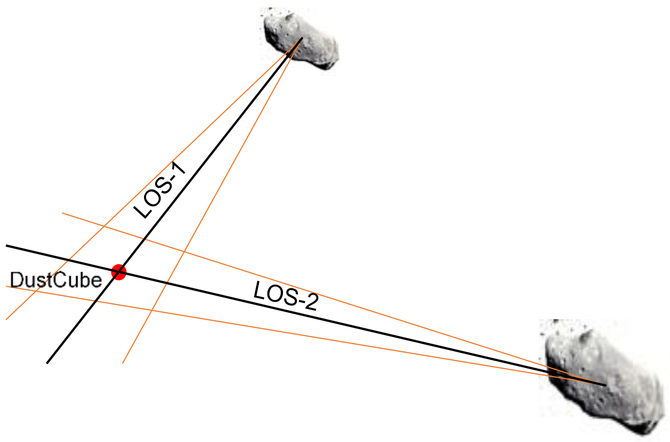


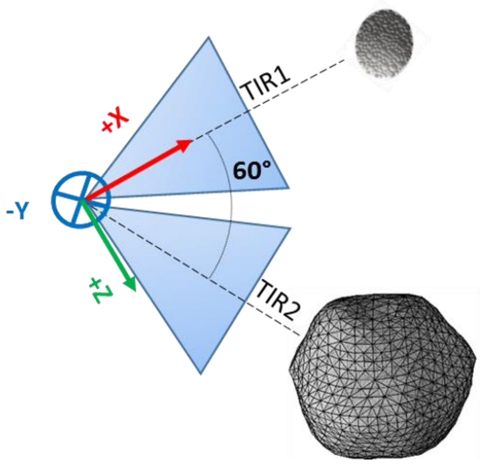
Mission Constraints:

- Maximum distance from Didymoon of 3 km, to perform in-situ measurements of the dust particles.
- Complex and highly uncertain dynamic environment.
- Minimum lifetime of 2 months.
- High level of autonomy, including navigation

OD Solution Approach (1)

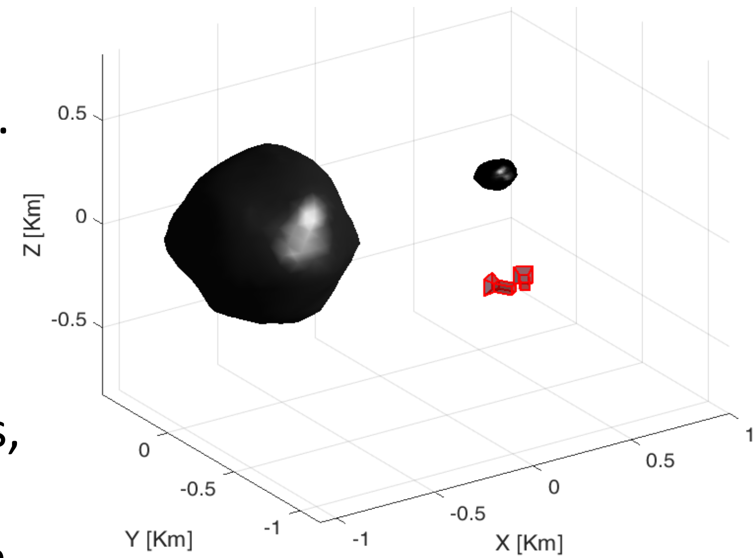
- The selected orbit determination (OD) strategy relies on the 3D localization concept:
  - observing two nearby objects whose relative position is known, the S/C position can be inferred with respect to the beacons.
- This approach is particularly suited in this case, given the binary nature of the Didymos system, where two close range beacons are in view.
- The 2 targets occupy a significant portion of the FoV: defining the LoS as the center of figure of each imaged target might lead to excessive errors.
- More complex techniques, such as those based on feature tracking, are not an option since we aim at real time operation onboard a CubeSat

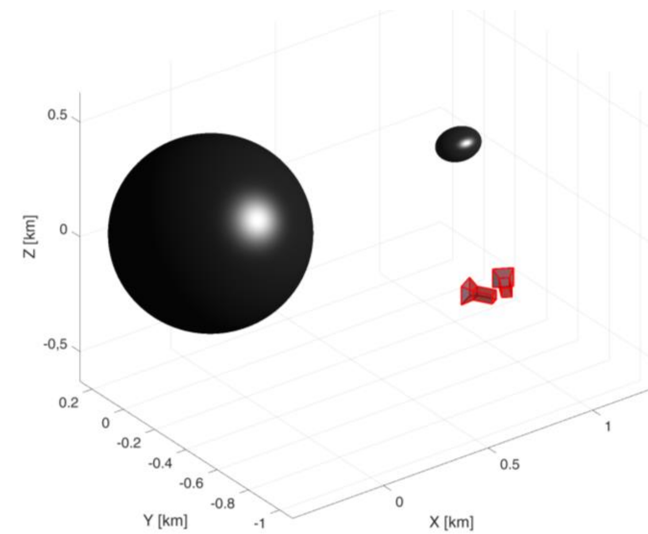




OD Solution Approach (2)

- We sought for some 3d localization making use of the knowledge of the asteroid shapes.
- We made use of some exact results that can be obtained when the imaged bodies are ellipsoids.
- By fitting ellipses to the detected limb points, the position can be expressed using the perspective mappings of an ellipsoid onto an ellipse in the image plane.
- Asteroids cannot be strictly considered ellipsoids, but one can seek for approximate position fixing *pretending* that the limb points where originated by some *equivalent* ellipsoids fitted to the actual asteroid shapes





The Math (1)

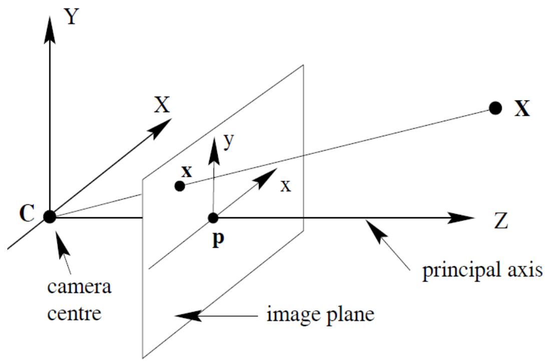
Theory based on a different derivation of existing results.

1. Given a *pinhole camera model*:

$$\mathbf{x}_i = \mathbf{KR}[\boldsymbol{I} \quad \boldsymbol{t}_w]\mathbf{X}_w = \mathbf{PX}_w$$

$\mathbf{x}_i$= image points; $\mathbf{X}_w$= 3d points; $\boldsymbol{t}_w$= position vector;

R = camera attitude;



R. Hartley, A. Zisserman, "Multiple View Geometry in Computer Vision", 2nd Ed. Cambridge University Press, 2004

2. and an *ellipsoid model*: $\mathbf{Q} = \begin{bmatrix} 1/a^2 & 0 & 0 & 0 \\ 0 & 1/b^2 & 0 & 0 \\ 0 & 0 & 1/c^2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_3 & \mathbf{0} \\ \mathbf{0} & -1 \end{bmatrix}$

3. The perspective projection of ellipsoids follows the mapping between *inverse* matrices:

$$\alpha \mathbf{C}^*_{im} = \mathbf{PQ}^*\mathbf{P}^T$$

$\mathbf{C}^*_{im}$= inverse fitted ellipse matrix (defined up to a scalar α

After some manipulations:

$$\boldsymbol{t}_w \boldsymbol{t}_w^T = \mathbf{Q}_3^* - \alpha \mathbf{R}^T \mathbf{C}^*_{im} \mathbf{R}$$

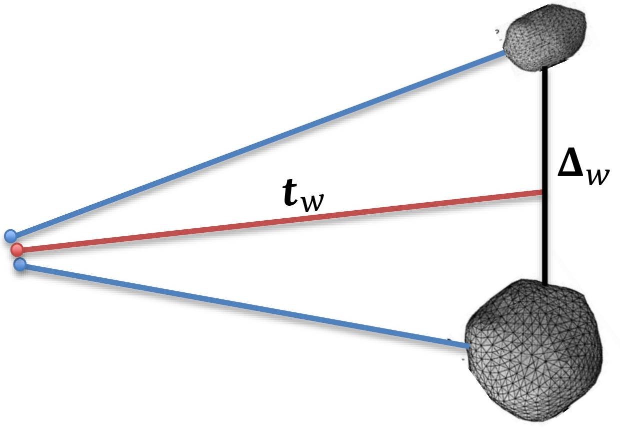
The Math (2)

LoS solution is computed as an eigenvector of matrix

$$\mathbf{R}^T \mathbf{C}_{im}^* \mathbf{R} \mathbf{Q}_3$$

If we now consider two targets, we can conceive two alternative formulations

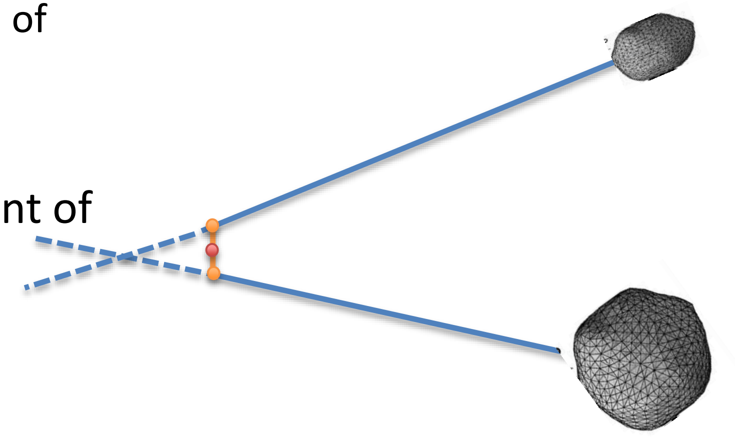
1. By summing up the two observation eqs one estimates the position towards the midpoint of the segment joining the two bodies



$$\boldsymbol{t}_w \boldsymbol{t}_w^T = 0.5\left[(\mathbf{Q}_1^* + \mathbf{Q}_2^*) - \mathbf{R}^T\left(\alpha \mathbf{C}_{im,1}^* + \beta \mathbf{C}_{im,2}^*\right)\mathbf{R}\right] - \boldsymbol{\Delta}_w \boldsymbol{\Delta}_w^T$$

Search for the best fit between the two observed target. The optimal (in a least squares sense) solution is given in terms of *singular value decomposition* of the rhs matrix.

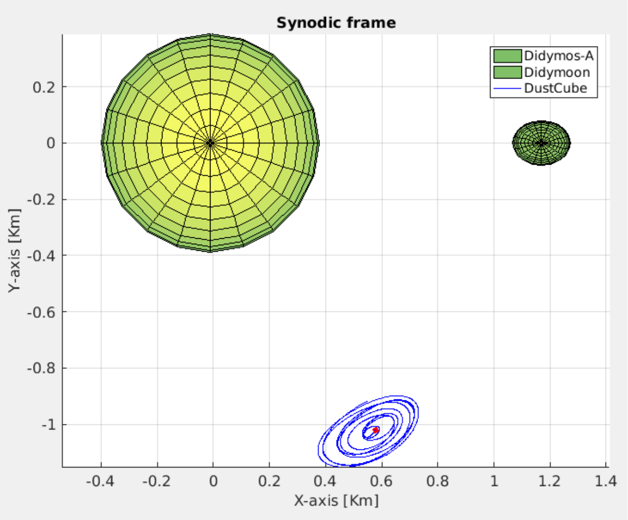
2. A LoS only solution, as the midpoint of the segment of closest approach



Simulation scenario @ L4 (1)

Representative DustCube trajectory, 1 week in the proximity of L4

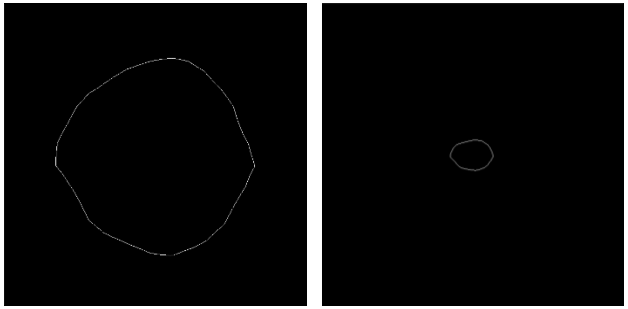
- Primary asteroid assumed to rotate about the orbit normal at 159.29 deg/h
- Secondary having a synchronous rotation around the primary
- Prescribed S/C attitude enables the cameras to follow the motion of the targets, imaged under some random off-nadir angles (σ=5°), with complete coverage over the trajectory.
- Errors in attitude knowledge (**R**) and asteroids relative position (**Δ**) also included



Error Source	Magnitude (1σ)	Notes
Absolute Knowledge Error (AKE)	0.1°	Attitude knowledge error, modelled as 3 successive normally distributed random rotations about the axes.
Misalignment Error (ME)	0.1°	Cameras’ alignment error, modelled as 3 successive normally distributed random rotations about the axes.
Asteroid Ephemerides	5 m	Error in the relative position of asteroids. Assumed conservatively higher than what expected after AIM radio science experiment

Simulation scenario @ L4 (2)

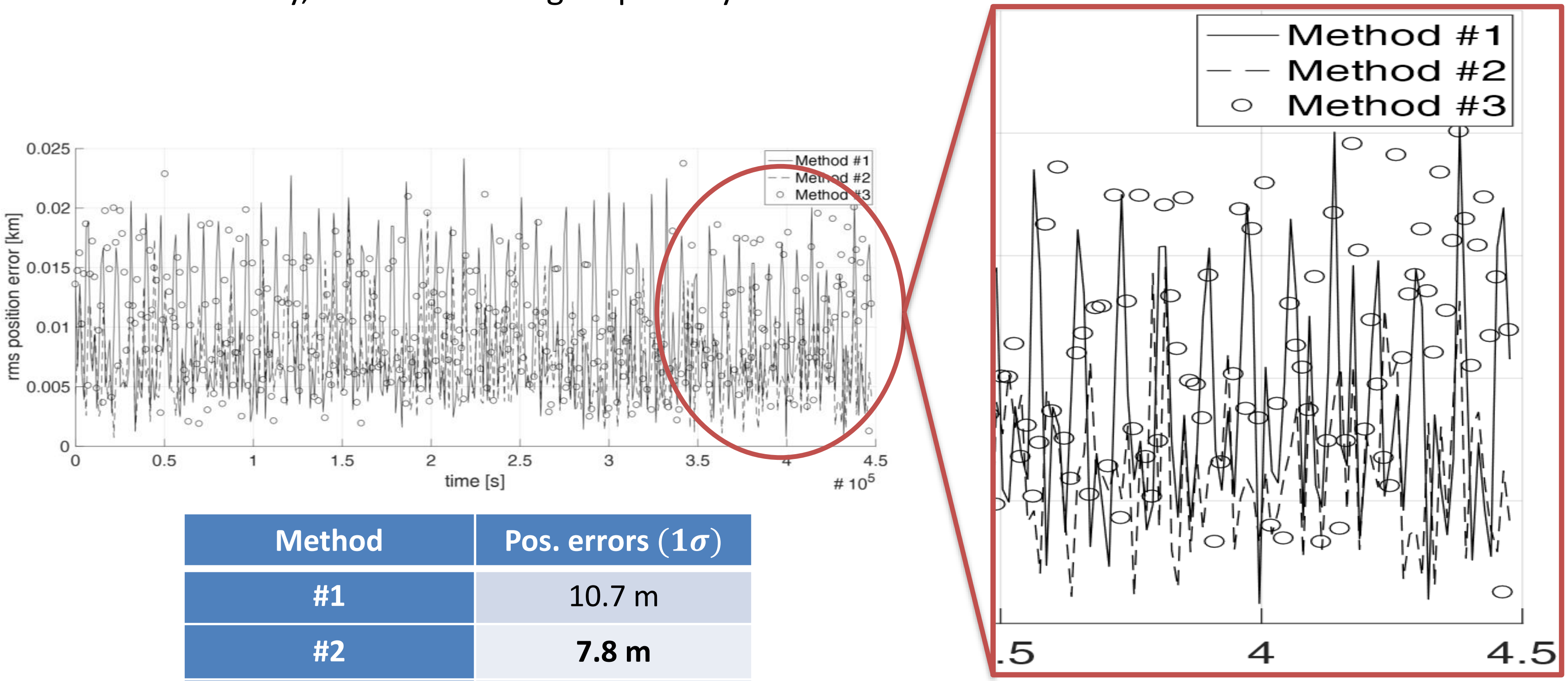
- At any point along the trajectory synthetic images of the target are generated.
- For each image pair:
  1. the limb of the targets is detected
  2. an ellipse is fitted to the detected limb points
- Position is then estimated with 3 methods:
  1. Least squares triangulation with LoS + apparent diameter from each target
  2. LoS only triangulation with LoS determined from the assumed shape
  3. LoS only triangulation with LoS determined from center of figure



Results

– Best performing algorithm is #2: LoS computed according to the equivalent ellipsoid model, with the full triangulation solution (method #1) performing worse.

- The reason is believed to be ascribed to the quite different size of the imaged bodies, which renders the error due to the ellipsoid approximation less pronounced for the smaller secondary, than for the largest primary



Method	Pos. errors (1σ)
#1	10.7 m
#2	**7.8 m**
#3	11.2 m

Computational burden

Two IR images of size 640x512px (≈ 0.33 Mpx) at ≈ 10^{-2} Hz

1. Image processing
   - Pre-processing (such as thresholding): 2×0.33 MFLOPs
   - Edge extraction (Sobel Convolution): 2×10 MFLOPs
   - Limb fitting (fit ellipse): 2×0.1 MFLOPs
2. Position solution (matrix inversion, eigenvalue and *svd* on 3x3 matrices):
   - # FLOPs is $O(n^3)$ → ≈ 10^4 FLOPs

Task	MFLOPs Image #1	MFLOPs Image #2
Thresholding	0.33	0.33
Edge Extraction	10	10
Limb Fitting	0.1	0.1
Position comp.	0.01	0.01
Total	**≈21**	

To be compared e.g. with LEON-2 rated @ 23 MFLOPs/sec indicates *feasible real time* operation.

Conclusions

- We investigated an approach to the autonomous optical navigation problem for a CubeSat within a binary asteroid system, taking as a baseline scenario 65803 Didymos.
  - We made use of some existing analytic solutions for the position determination when imaging ellipsoids, which we extended to treat multiple targets.
  - A formulation was developed with S/C position best-fitted between the LoS plus range that are obtained from each target.
- A simulation scenario was set up and alternative formulations for the position solution compared.
  - Results shows that the target accuracy of ***10m rms*** can be met with a LoS based navigation with the centroid computed according to the assumed asteroids shapes.
- The estimated overall MFLOPs budget suggests for a feasible real-time, on-board implementation on a CubeSat platform.

Acknowledgments

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