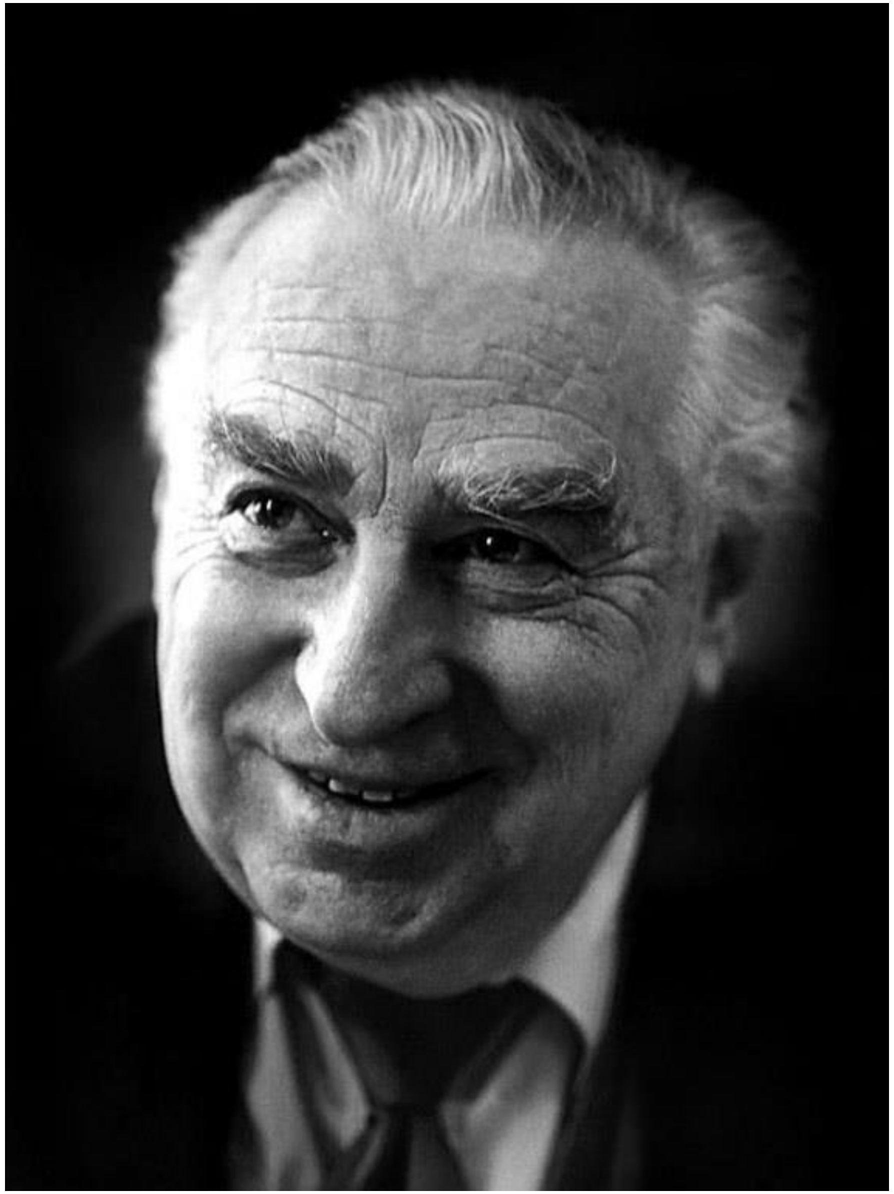
From Classical Theory to Innovation Practice. In memory of Professor Vladimir Beletsky

Mikhail Ovchinnikov

Keldysh Institute of Applied Mathematics of RAS,

Vladimir Beletsky
(2.05.1930 – 20.07.2017)



Professor,

Doctor of Science,

Corresponding Member of the Russian Academy of Sciences,

Member of the International Academy of Astronautics

Principal Researcher at the Keldysh Institute of Applied Mathematics of RAS

Professor of the Moscow State University

Outstanding researcher and discoverer

Branches where Professor made great input and left footprints:

- attitude dynamics of a rigid body in the central Newtonian gravitational and geomagnetic fields
- satellite attitude motion determination with sensors measurements processing
- interorbital optimum transfers by low thrust
- Formation Flying precursor
- resonant rotation of the planets
- tether systems dynamics
- bipedal gait robot dynamics
- poetry, essay, spaceflight dynamics polarization

However, before those achievements at 12 after grave illness he became deaf…

Footsteps on the Earth

- He was born in Irkutsk City on Baykal Lake,
- graduated from secondary school with distinction in Smolensk City,
- graduated from the Moscow State University with distinction,
- entered the Applied Mathematics Department at the Steklov Math Institute headed by Mstislav Keldysh, later transformed to the Institute of Applied Mathematics of USSR Academy of Sciences
- next there was life in science, achievements, trips, PhD students supervising …

Other but principal indications
(sketches drawn by G. Efimov for Prof’s 80th Anniversary)

“Capitoline Wolf”

Beletsky’s books

V.V. Beletsky, Spacecraft Attitude Motion in the Gravitational Field. Moscow, Moscow State University, 1975 (in Russian).

V.V. Beletsky, Motion of an Artificial Satellite about its Center of Mass. Moscow, "Nauka", 1965 (in Russian), Israel Program for Scientific Translation, Jerusalem, 1966 (In English).

V.V. Beletsky, A.M. Yanshin, The Influence of Aerodynamics Forces on Spacecraft Rotation. Kiev. "Naukova Dumka", 1984 (in Russian).

V.V. Beletsky. A.A. Khentov, Magnetized Spacecraft Attitude Motion. Moscow. "Nauka", 1985 (in Russian).

V.V. Beletsky, E.M. Levin, Dynamics of Space tether systems. Moscow. "Nauka", 1990 (in Russian). AAS Publ., Univelt Inc., San Diego, 1993 (in English).

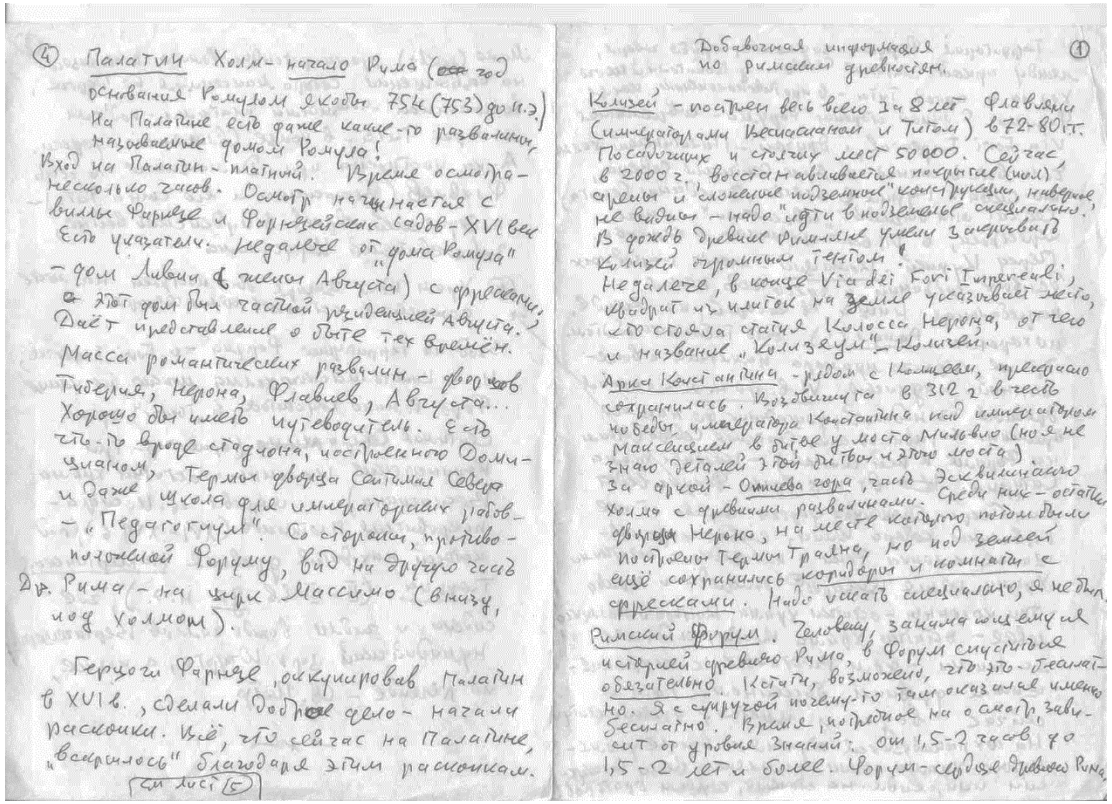
V.V. Beletsky. A.A. Khentov, Resonance Rotations of Celestial Bodies. Nizhniy Novgorod, 1995 (in Russian).

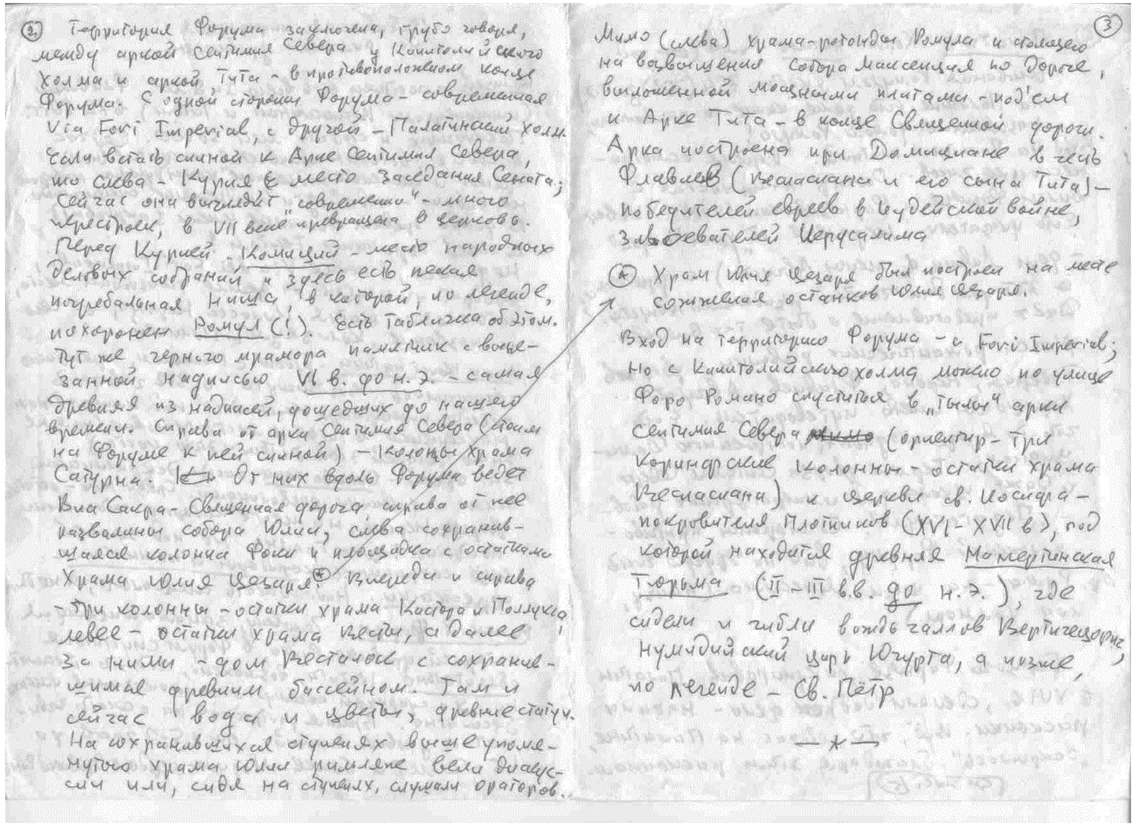
V.V. Beletsky, Esseys on Space Bodies Motion. Moscow. “Nauka”, 1972 (in Russian), "Birkhauser", Basel, 2001 (in English).

V.V. Beletsky, Six dozens, Moscow; Izhevsk, 2004 (in Russian).

V.V. Beletsky, Out of fortune, Moscow, 1993 (in Russian).

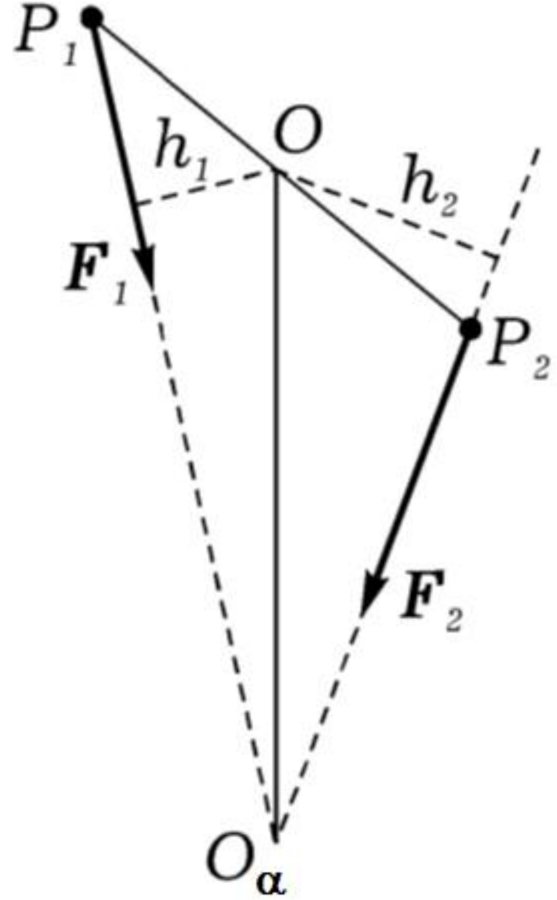
He loved Italy and Italy loves him





"… to explore the Roman Forum it is required from hours to years… It depends on a person."

Attitude dynamics of a rigid body in the central Newtonian gravitational fields



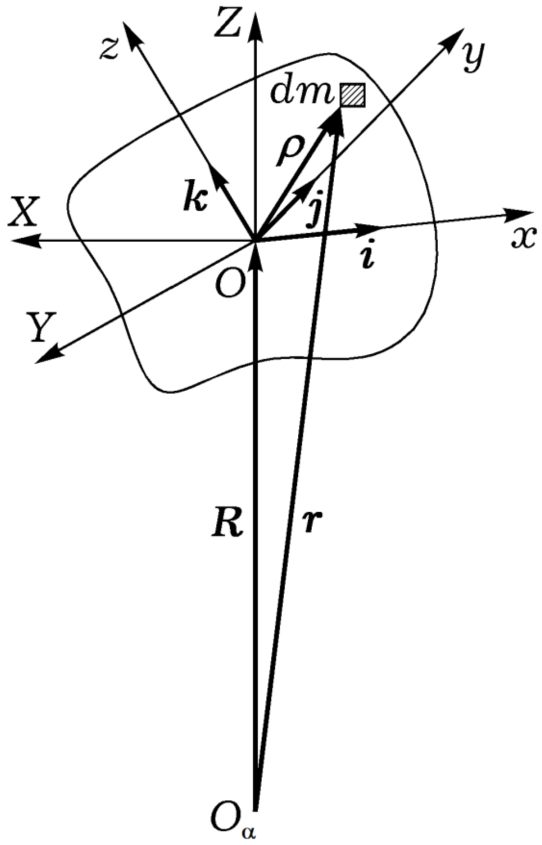
On the body moving around the Earth the gravity-gradient torque acts:

two-particle approximation

rigid body

$$\mathbf{M}_0 = \frac{3\mu_g}{R^3}\mathbf{E}_3 \times \mathbf{J}\mathbf{E}_3$$

$$\mathbf{J} = diag\{A, B, C\}$$



Beletsky’s Inequalities [1956]

In a circular Keplerian orbit three-axis rigid body attains 24 equilibria w.r.t. the Orbital Reference Frame.

Beletsky obtained necessary and sufficient conditions

$$B > A > C$$

of equilibria stability.

Only 4 various equilibria satisfy these inequalities (the Moon occupies one of them)

To obatain these inequalities there were derived first integral (The Jacobi Integral)

$$\frac{1}{2}(A\overline{\omega}_1^2 + B\overline{\omega}_2^2 + C\overline{\omega}_3^2) + \frac{3}{2}\omega_0^2(Aa_{31}^2 + Ba_{32}^2 + Ca_{33}^2) - \frac{1}{2}\omega_0^2(Aa_{21}^2 + Ba_{22}^2 + Ca_{23}^2) = h_0$$

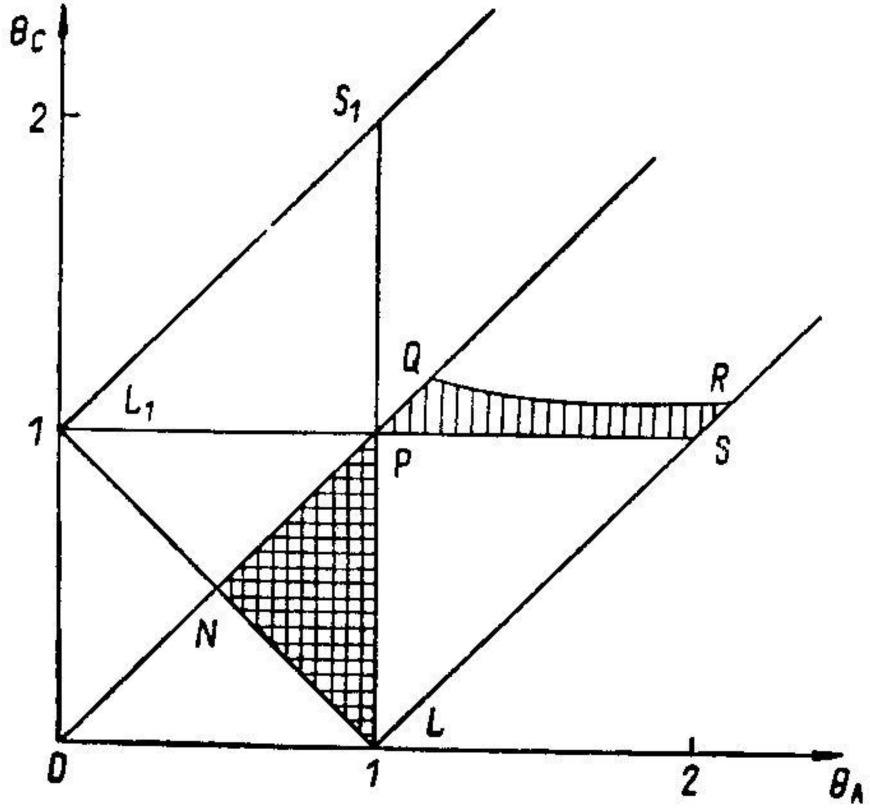
and linearized equations

$$B\ddot{\alpha} + 3(A-C)\omega_0^2\alpha = 0,$$
$$C\ddot{\beta} - (A-B+C)\omega_0\dot{\gamma} + (B-A)\omega_0^2\beta = 0,$$
$$A\ddot{\gamma} + (A-B+C)\omega_0\dot{\beta} + 4(B-C)\omega_0^2\gamma = 0$$

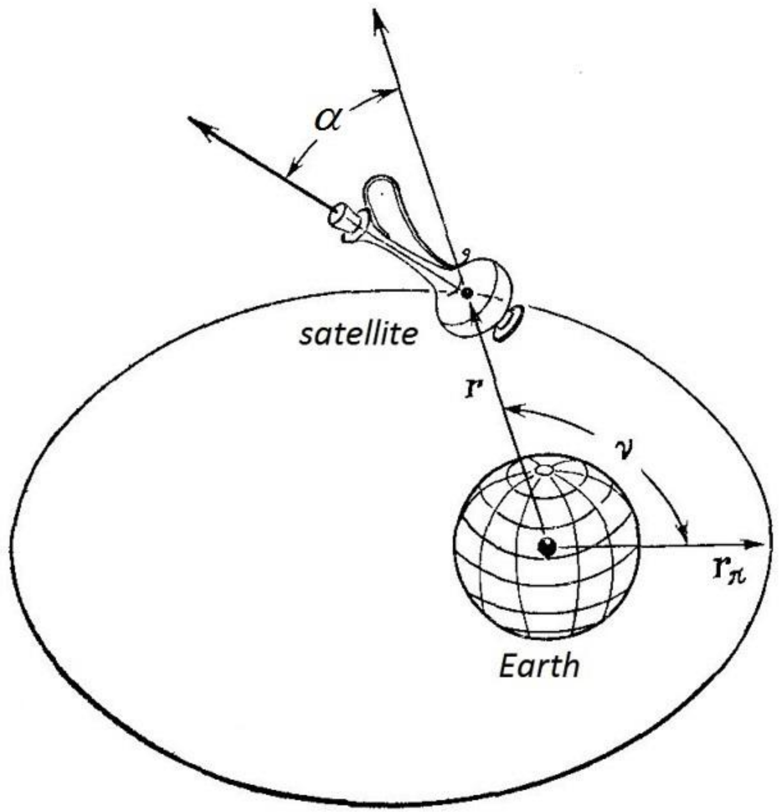
with corresponding secular equation w.r.t. ρ

$$\theta_A\theta_C\rho^4 - [\theta_A\theta_C + 3\theta_C(1-\theta_C) + (1-\theta_A)(1-\theta_C)]\rho^2 + 4(1-\theta_A)(1-\theta_C) = 0$$

Finally, the inequalities are shown in the graph



Planar motion in the elliptical orbit*



* Such style playful pictures were drawn by Professor Igor Novozhilov, friend of Beletsky and are taken from ***Esseys on Space Bodies Motion*** "

Beletsky’s Equation [1959]

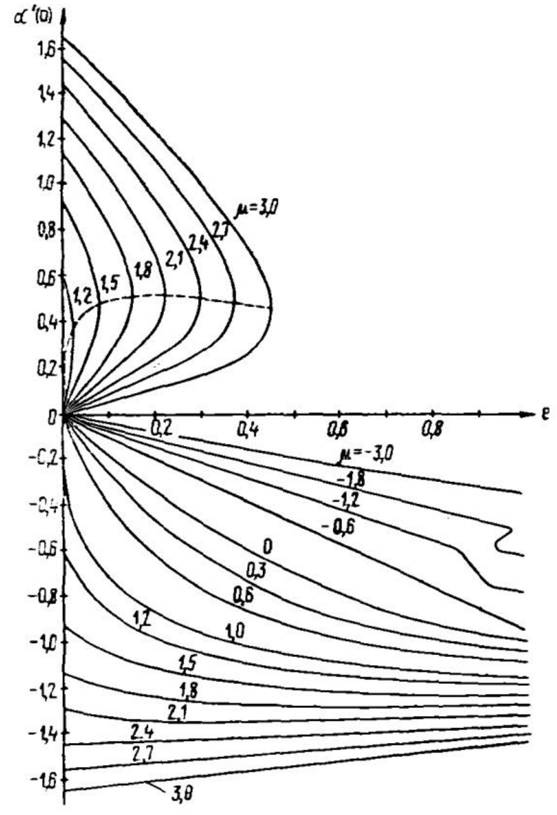
The planar rotational motion of a rigid body in elliptical orbit is described by the equation

$$\alpha''(1+e\cos\upsilon)-2e\alpha'\sin\upsilon+\mu\sin\alpha\cos\alpha=2e\sin\upsilon$$

where υ is a true anomaly, (‘) denotes derivation w.r.t. to υ, α is a pitch angle, e is an orbit eccentricity, μ=(A-C)/B is an inertial parameter

Asymptotical and numerical 2π – periodical solutions of the equation were obtained by V. Sarychev

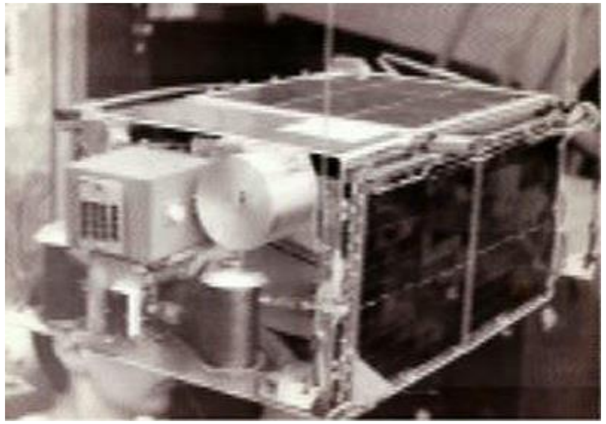
$$\alpha = \left[\frac{2e}{\mu-1} + \frac{4\mu e^3}{(\mu-1)^4} + \dots\right]\sin\upsilon + \left[\frac{3e^2}{(\mu-1)(\mu-4)} + \dots\right]\sin 2\upsilon +$$
$$\left[\frac{4(10\mu^2-22\mu+9)3e^3}{3(\mu-1)^3(\mu-4)(\mu-9)} + \dots\right]\sin 3\upsilon + \dots$$



Bifurcation curve built by Beletsky separates manifolds
with single and three periodical solution

$$e = \left(\frac{2}{3}\right)^{3/2} \frac{(\mu-1)^{3/2}}{2\sqrt{\mu}}$$

Gravity-Gradient ACS developed for small satellites by our team



Iskra-1 (RK-01), 1981, launched with Meteor-Priroda as a piggy-back

US-Russian REFLECTOR, 6 kg and Pakistanean BADR-B, 70 kg, 2001, launched together with Meteor as a piggy-back

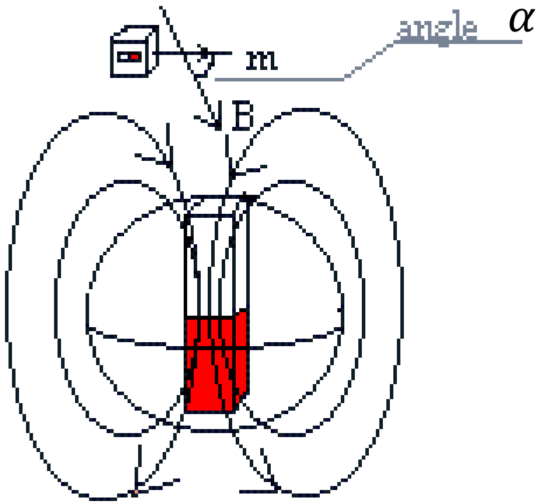
… and studied …

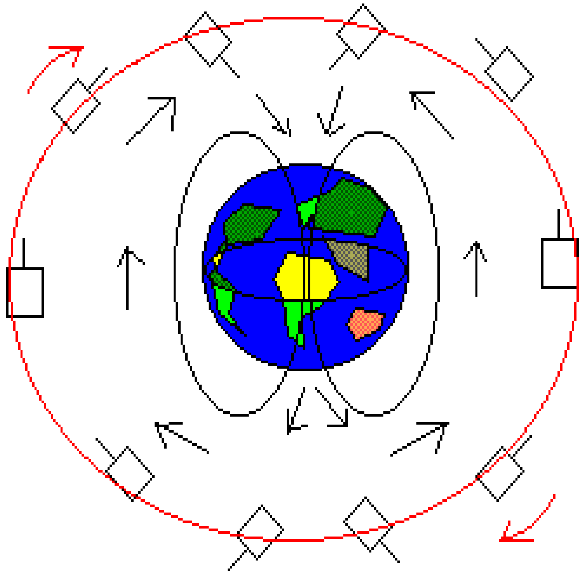
… MIR OS and ISS

Salyut Orbital Stations (Salyut-7 reanimation !)

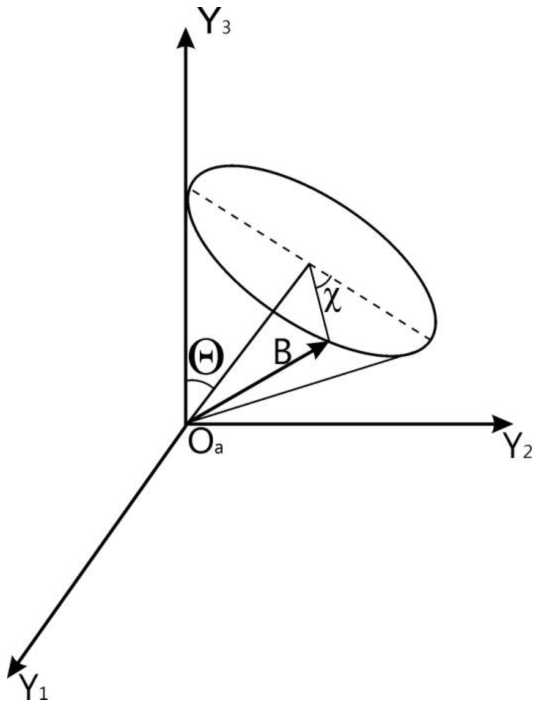
Attitude motion in the Geomagnetic field (Beletsky and Khentov studies)

assumptions: direct dipole for approximation of GF, satellite is an axisymmetrical rigid body with a permanent magnet along with the axis of symmetry





Representation of a spatial motion



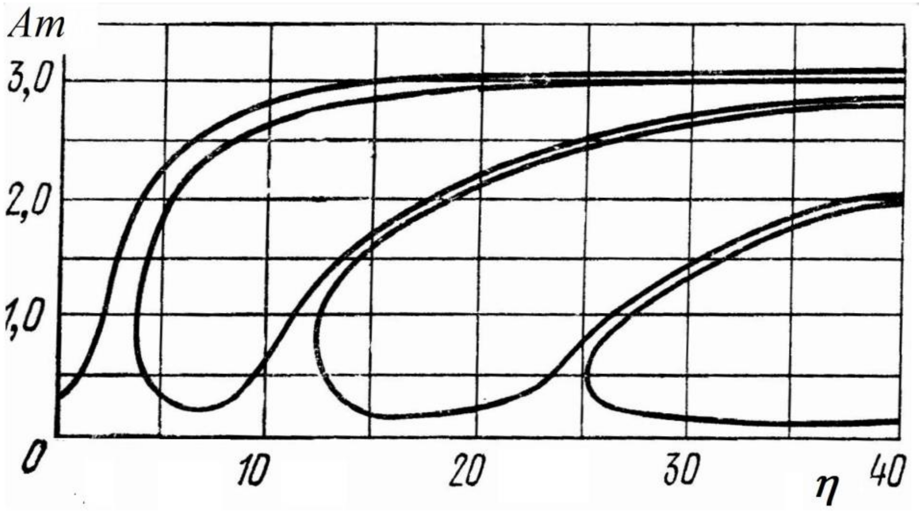
$$\operatorname{tg}\Theta = \frac{3\sin 2i}{2\left(1 - 3\sin^2 i + \sqrt{1 + 3\sin^2 i}\right)}$$

*P.Donoho from Bell Telephone Laboratory, reference in E. Zajac's paper [1962]

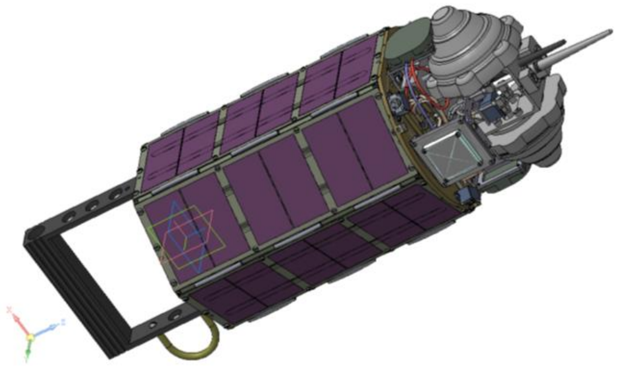
Beletsky derived equation of a planar motion

$$\alpha'' + \eta\sqrt{1+3\sin^2 u}\,\sin\alpha = \frac{6\sin 2u}{(1+3\sin^2 u)^2}$$

that immediately showed
that there no equilibria
w.r.t. vector of GM induction
but there external and parametric
resonances due to time-depend terms

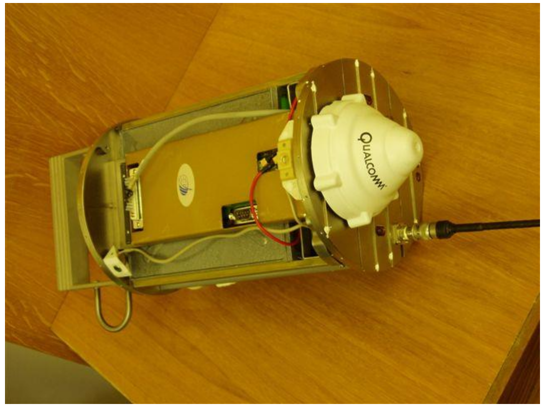


Magnetic (passive) ACS for small sats developed

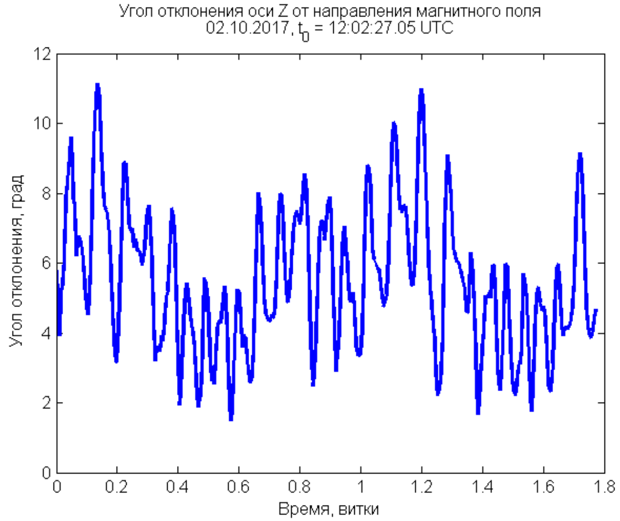


Russian TNS-0 #2, 5 kg, launched in 2017

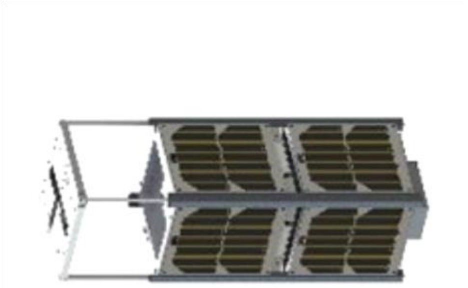
Swedish Munin, 6 kg, launched in 2000 (credid: IRF)



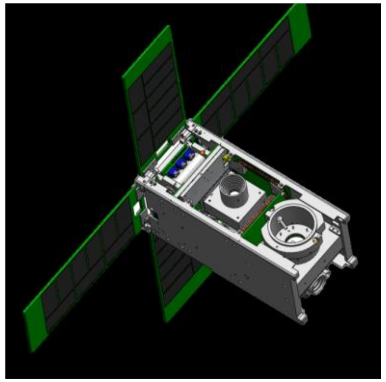
Russian TNS-0 #1, 5 kg launched in 2005



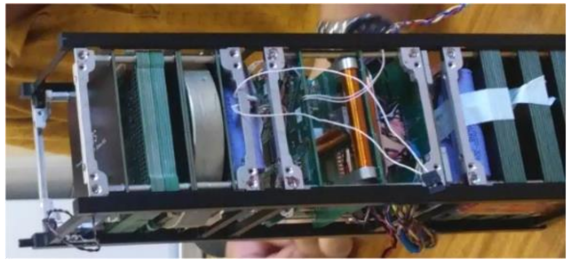
Magnetic (active) ACS for small sats developed



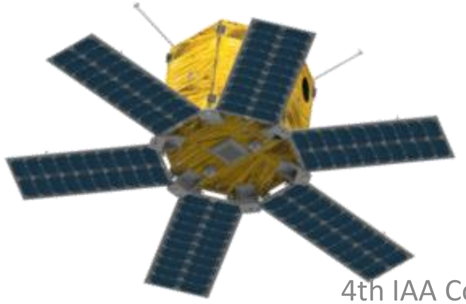
Russian SamSat-QB-50, 2U, passive aerodynamical ACS



US CXBN-2, 3U, launched in 2017

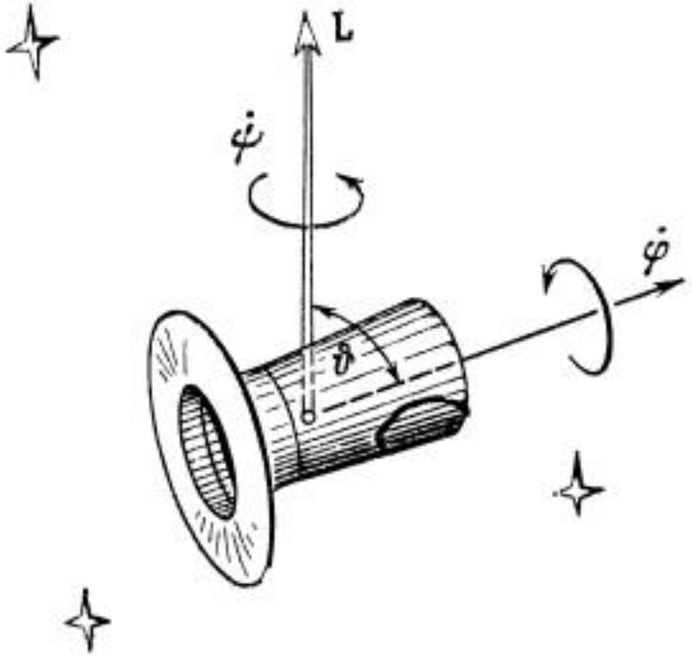


Russian AIS-CubeSat, 3 kg, JSC RSS



Chibis-M (IKI RAS, 43 kg, 2012), TabletSat-Aurora (SputniX, 28 kg, 2014)

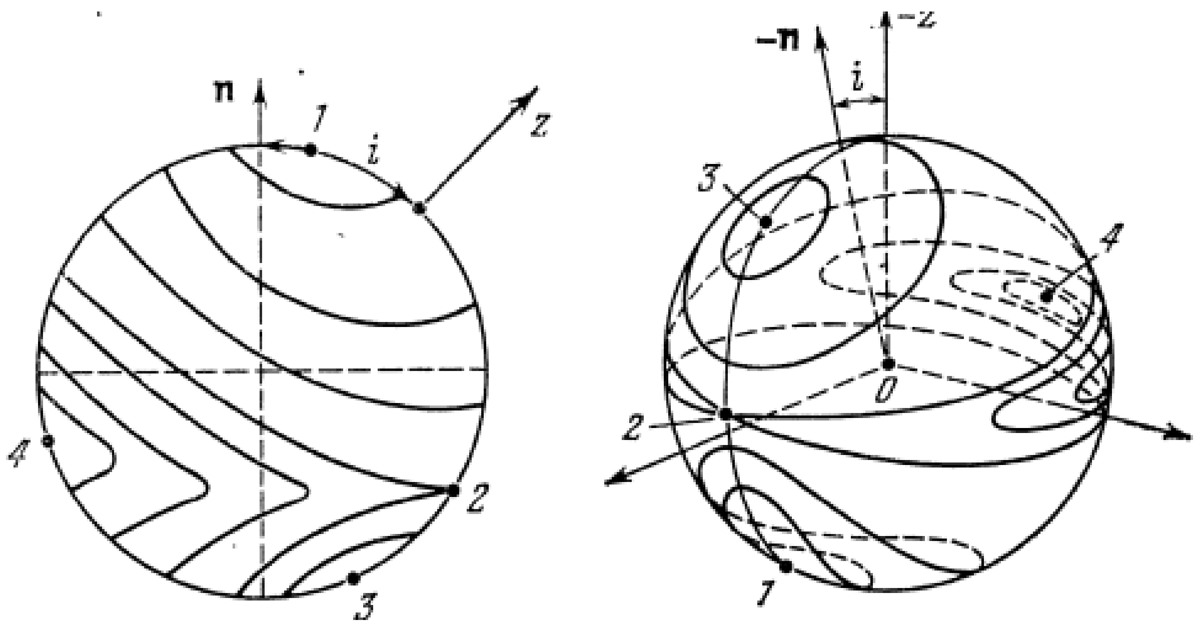
Beletsky’s “Garden” and his “Flowers”



$$\frac{dL}{dt} = \frac{\partial U}{\partial \psi}, \quad \frac{d\sigma}{dt} = \frac{1}{L \sin \rho} \frac{\partial U}{\partial \rho},$$
$$\frac{d\rho}{dt} = \frac{1}{L \sin \rho} \left(\frac{\partial U}{\partial \psi} \cos \rho - \frac{\partial U}{\partial \sigma} \right),$$
$$\frac{d\psi}{dt} = \frac{L}{A} - \frac{1}{L} \left(\frac{\partial U}{\partial \rho} \operatorname{ctg} \rho + \frac{\partial U}{\partial \vartheta} \operatorname{ctg} \vartheta \right),$$
$$\cos \vartheta = C r_0 / L.$$

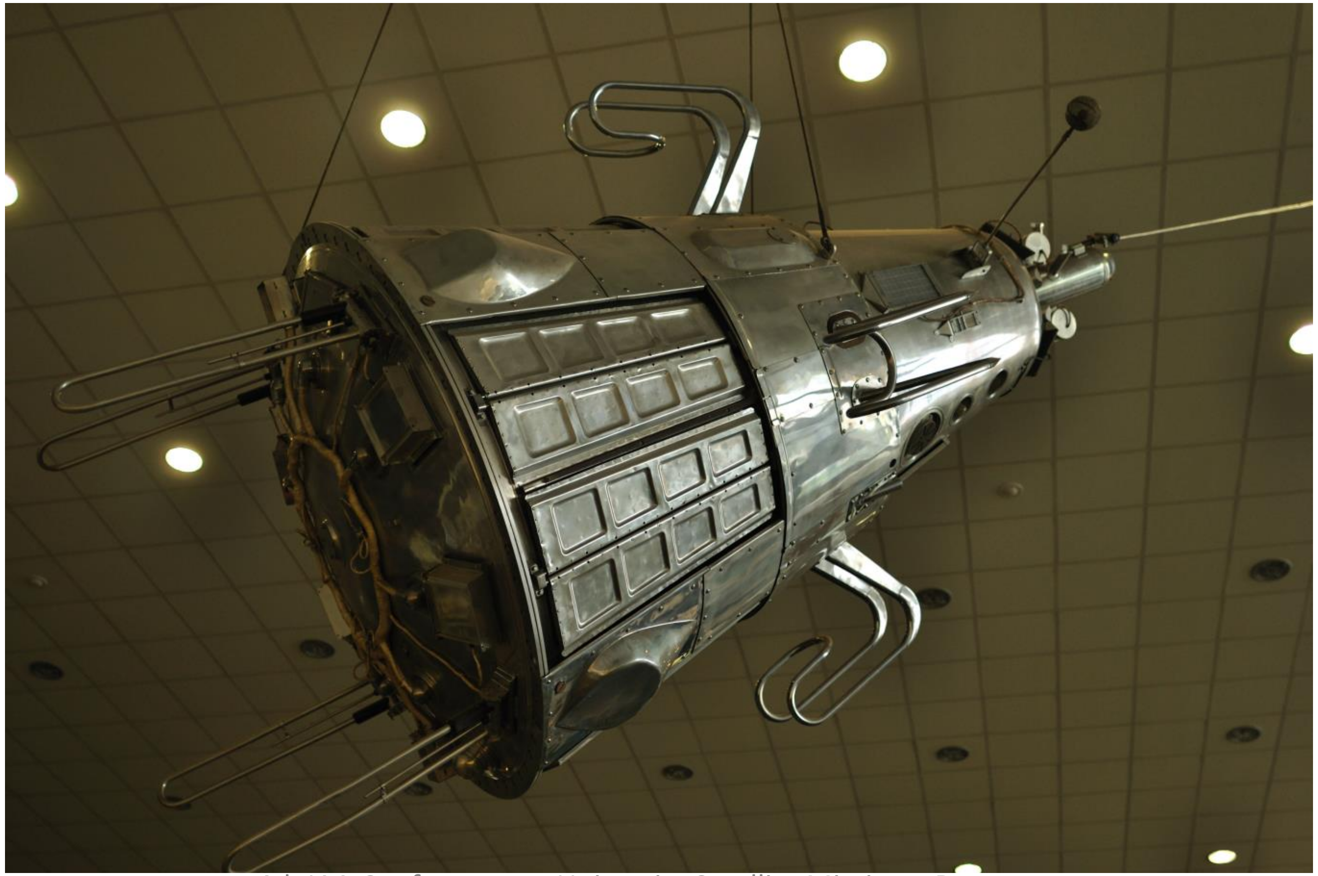
Averaging method is his shovel

Interpretation of the US Pegasus satellite evolution due to orbit precession



Momentum vector hodograph

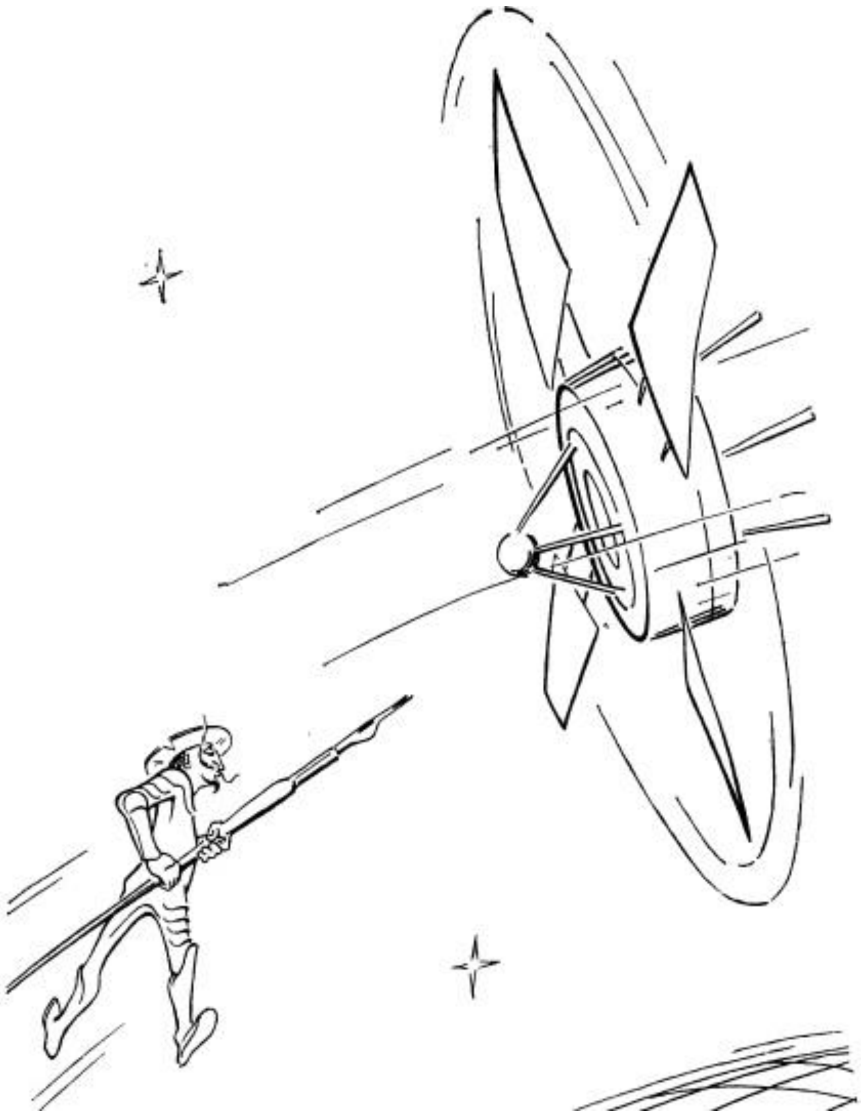
The Third Soviet Satellite (15th of May, 1958)



Attitude motion determination

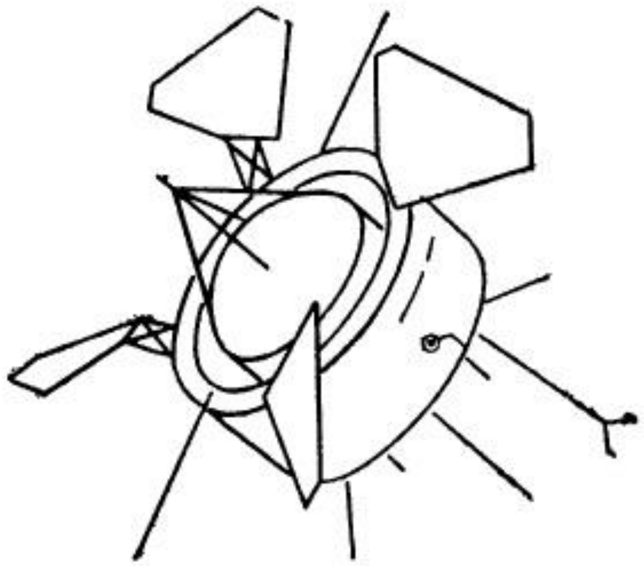
- The problem of attitude determination was formulated by Beletsky in 1958* within the launch campaign of the Third Soviet Satellite
- He implemented the Least Mean Square Method for determination of the initial conditions of the angular motion with respect to the inertial space and unknown precisely external disturbances at the given interval of time
- The measurements of the on-board magnetometer were used

*V.V. Beletsky, Yu.V. Zonov, Rotation and Orientation of the Third Soviet Satellite, Artificial Satellites of the Earth Digest, 1961, Issue 7 (in Russian)



More specific study was done by Beletsky for heavy Proton satellite

Propeller composed by the solar arrays tricky effected the angular motion due to atmosphere resistance

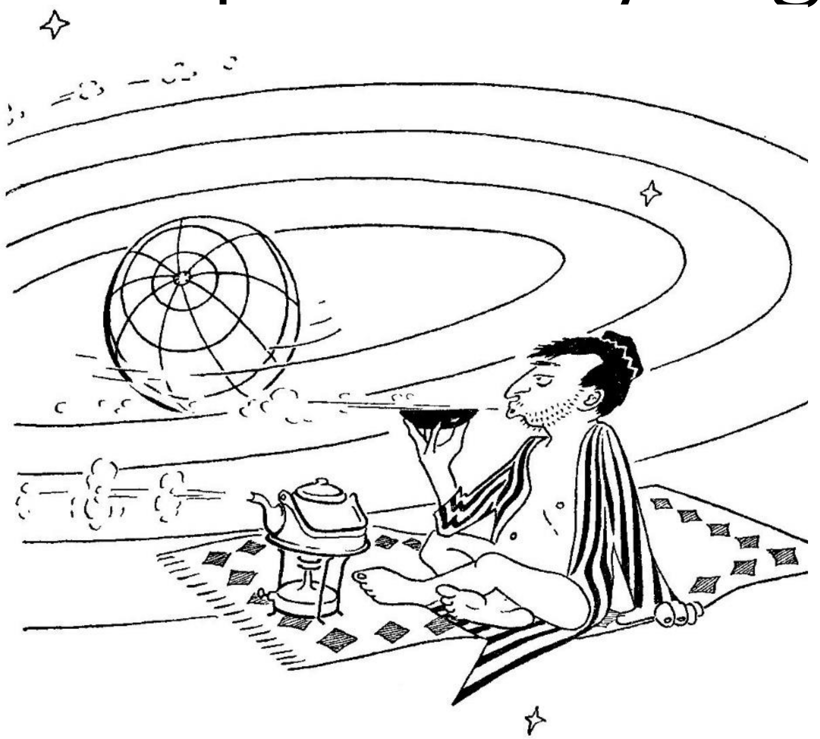


«Precursor» of Formation Flying and tether systems

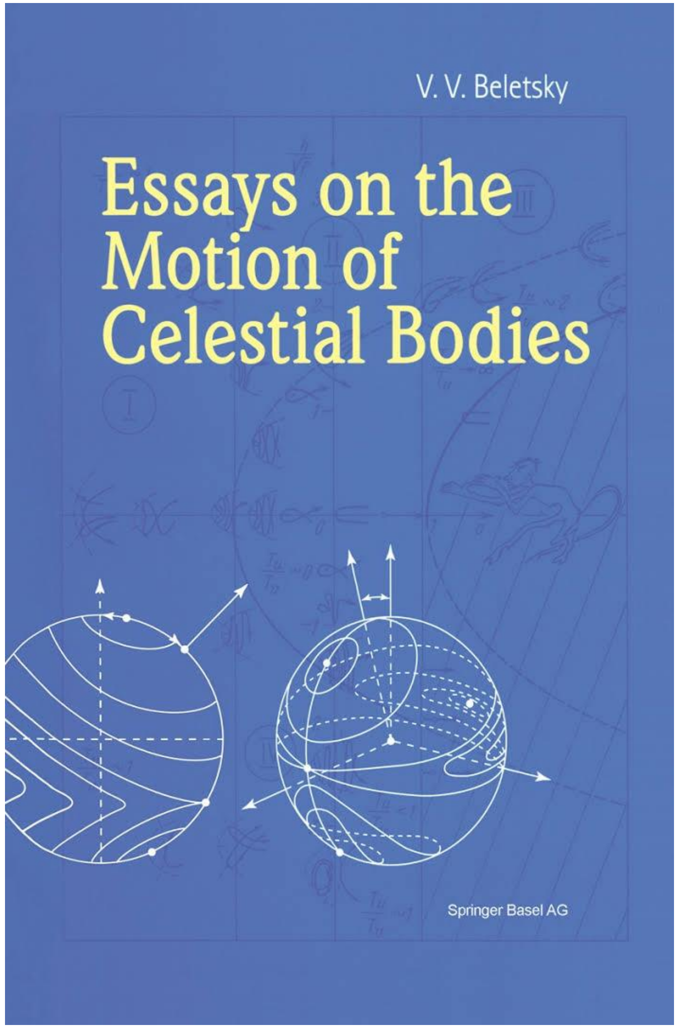
On March, 1965 cosmonaut Alexey Leonov exited Voskhod-2 SC and threw out a cap of camera to the Earth. Also, he was connected with SC by elastic cord

Beletsky studies the relative motion of the cap and introduced systems with bilateral constraints

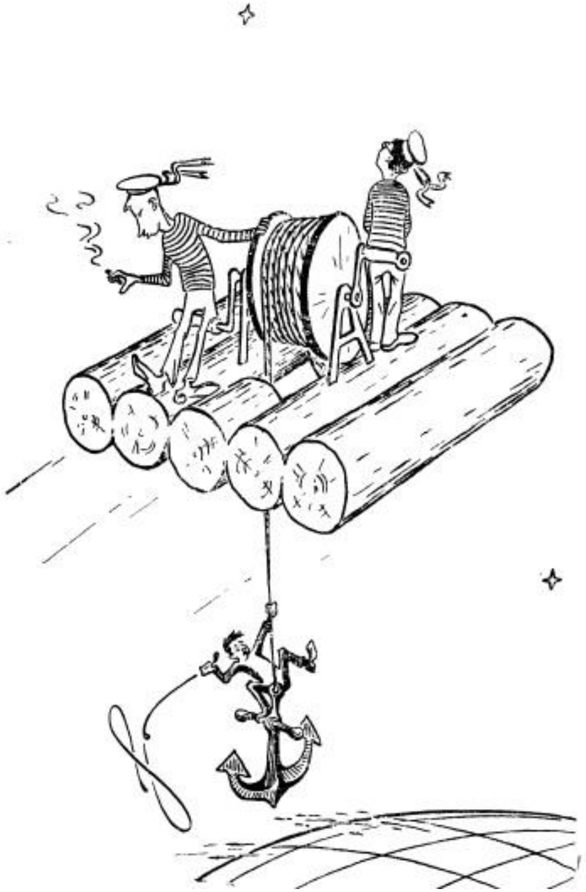
Low-thrust interplanetary flights



To solve the problem of optimal control minimizing the consumption of fuel Beletsky used Pontryagin’s Maximum Principal
To solve the BVP he introduced the Transport Trajectory as a first approximation


V. V. Beletsky
Essays on the Motion of Celestial Bodies
Springer Basel AG

Gravicraft



A wide variety of tasks and scientific problems solved by Beletsky are associated with bouquet

Original vs. copy

He loved life and liked to say “coffin does not have a roof rack” …

Aknowledgement

The paper is supported by the Russian Science Foundation (project # 17-71-20117)