IAF Astrodynamics Committee
TECHNICAL KEYNOTES IAC2017

23rd John V. Breakwell Memorial Lecture

APPLIED ASTRODYNAMICS:
FROM DYADICS TO UNIVERSITY SATELLITES

Filippo Graziani

Adelaide, 27 September 2017

Memories

Professor John V. Breakwell
Durand Building, Stanford (1975)

Professor John V. Breakwell and myself
30th IAC, Munich, Germany (1979)

John V. Breakwell Teaching

- A special teacher
- A special scientist
- A special man

"An empty desk is a sign of an empty mind. JVB"

- A tool I learnt to use under his direction is the dyadic.
- Dyadics allow to simplify our approach to complex problems, as I always attempted to do.
- I would start this talk by discussing their relevance in Astrodynamics by means of some examples I used during my lectures.

Dyads and Dyadics: a reminder

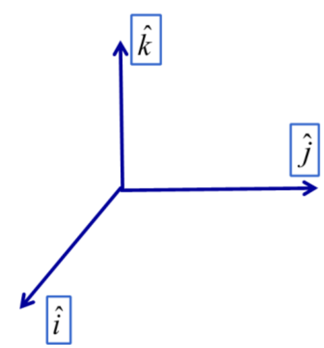
A **dyad** is a couple of vectors side by side.
In mathematical words, it can be represented as a matrix.

$$\underline{\underline{d}} = \vec{a}\vec{b} \Rightarrow \underline{\underline{d}} \cdot \vec{u} = \vec{a}\left(\vec{b} \cdot \vec{u}\right)$$

A **dyadic** is a linear combination of dyads.

A unity dyadic is an identity operator

$$\underline{\underline{u}} = \hat{i}\hat{i} + \hat{j}\hat{j} + \hat{k}\hat{k}$$



that is

$$\underline{\underline{u}}\,\vec{a} = \left(\hat{i}\hat{i} + \hat{j}\hat{j} + \hat{k}\hat{k}\right) \cdot \vec{a} = \left(\hat{i} \cdot \vec{a}\right)\hat{i} + \left(\hat{j} \cdot \vec{a}\right)\hat{j} + \left(\hat{k} \cdot \vec{a}\right)\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \vec{a}$$

Usefulness of Dyadics

- As it was pointed out by D. DeBra, dyadics help to separate a problem into two phases:
  1) **The thinking phase;**
  2) **The numerical evaluation phase.**

- During the thinking phase, the notation is compact and helps to retain a clearer interpretation.

- These methods also prepare the problem for the numerical evaluation phase since they easily permit to transform the dyadic and polyadic notation into coordinates convenient for digital programming.

- Perhaps the most remarkable simplification is achieved when the effect of a perturbation is considered. A consistent work saving is achieved through cancellation of the unperturbed solution in vector form.

3) a third phase, i.e. the experimental one, will be added later.

Dyadics in Astrodynamics

Some Astrodynamics applications where dyadics play an important role:

ORBITAL DYADIC

Wobble due to J2

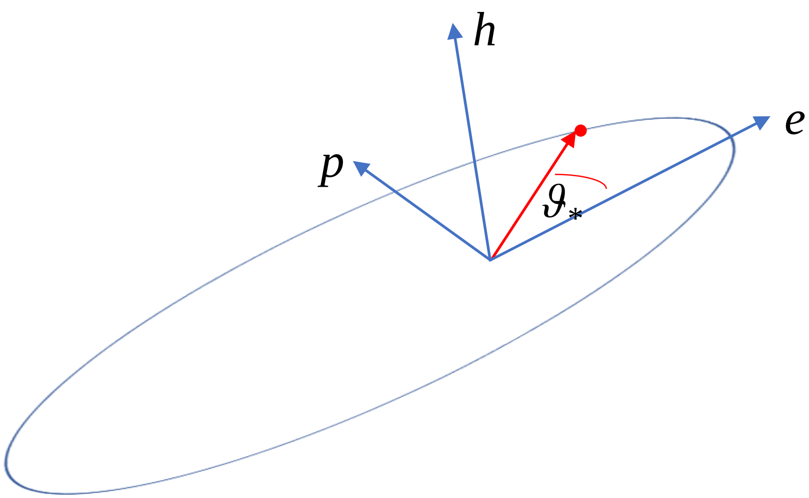
GRAVITY GRADIENT DYADIC

Third body perturbation
Wobble due to Sun and Moon
Tidal effects
Gravity Gradient Stabilization
Magnetic Torque Formulation
Proximity Operations
Restricted three bodies problem formulation
Sphere of Influence Computation
Extra Vehicular Activities
Launcher Optimal Guidance
Weak Stability Boundary Transfers
Structural Stability under GG compression

INERTIA DYADIC

Gravity Gradient Stabilization
Tethered Systems

Orbital Average of $\hat{r}\hat{r}$ in the Apsidal Frame



$$\hat{r} = \hat{e}\cos\vartheta_* + \hat{p}\sin\vartheta_*$$

$$\hat{r}\hat{r} = \hat{e}\hat{e}\cos^2\vartheta_* + \hat{p}\hat{p}\sin^2\vartheta_* + (\hat{e}\hat{p} + \hat{p}\hat{e})\cos\vartheta_*\sin\vartheta_*$$

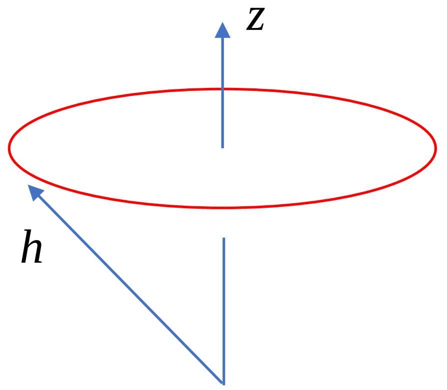
The average in ϑ_* will be

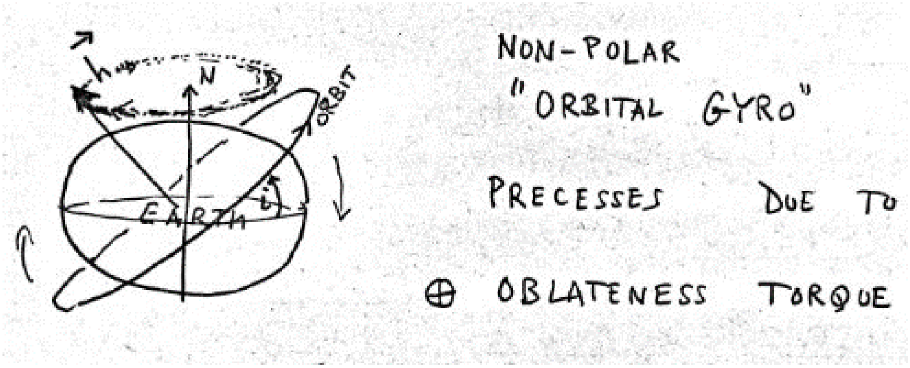
$$\langle\hat{r}\hat{r}\rangle = \frac{1}{2}(\hat{e}\hat{e} + \hat{p}\hat{p}) = \frac{1}{2}\left(\underline{\underline{u}} - \hat{h}\hat{h}\right)$$

Wobble due to Earth Oblateness

Orbital - Gyro Equivalence (*W. Kaula*)

A satellite orbiting a massive body may be regarded as a gyroscope whose spin axis is the orbit angular momentum and whose mass is uniformly distributed along the orbit as a ring.





An original sketch by J.V. Breakwell

Analogy to Poisson rule:

$$\frac{d\vec{h}}{dt} = \dot{\vec{h}} + \vec{\omega} \times \vec{h} \qquad \vec{\omega} = -\frac{3}{2} J_2 \frac{R_\oplus^2}{p^2} \hat{z} \cdot \hat{h}\, \hat{z} \qquad \vec{\omega} \approx 4.8^\circ / year$$

(for GEO)

Wobble due to Earth Oblateness

From Earth quadrupole moment
$$U_{J_2} = J_2 \mu \frac{R_\oplus^2}{r^3}\left[\frac{1-3(\hat{r}\cdot\hat{z})^2}{2}\right]$$

The J2 perturbation in vectorial form is
$$\vec{f}_{J_2} = -\frac{3}{2} J_2 \frac{\mu R_\oplus^2}{r^4}\left\{\left[1-5(\hat{r}\cdot\hat{z})^2\right]\hat{r} + 2(\hat{r}\cdot\hat{z})\hat{z}\right\}$$

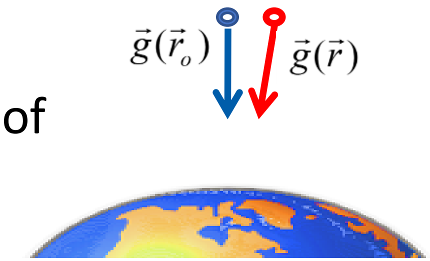
The effect is
$$\frac{d\vec{h}}{dt} = \vec{r}\times\vec{f}_{J_2} = -3J_2 \frac{\mu R_\oplus^2}{r^4}\hat{r}\cdot\hat{z}\ \vec{r}\times\hat{z}$$

$$\frac{d\vec{h}}{d\vartheta} = -3J_2 \frac{\mu R_\oplus^2}{r^4}\hat{r}\cdot\hat{z}\ \vec{r}\times\hat{z}\left(\frac{r^2}{h}\right) = -3J_2\left(\frac{R_\oplus}{p}\right)^2 h(1+e\cos\vartheta_*)\hat{z}\cdot\hat{r}\ \hat{r}\times\hat{z}$$

$$\left\langle\frac{d\vec{h}}{d\vartheta}\right\rangle = -\frac{3}{2}J_2\frac{R_\oplus^2}{p^2}h\hat{z}\cdot\left(\underline{\underline{u}}-\hat{h}\hat{h}\right)\times\hat{z} = \frac{3}{2}J_2\frac{R_\oplus^2}{p^2}h\hat{z}\cdot\hat{h}\hat{h}\times\hat{z}$$

$$\vec{\omega} = -\frac{3}{2}J_2\frac{R_\oplus^2}{p^2}\hat{z}\cdot\hat{h}\ \hat{z}$$

Gravity-Gradient Dyadic



The Gravity Gradient represents the linearized difference of the gravity acceleration in two different points

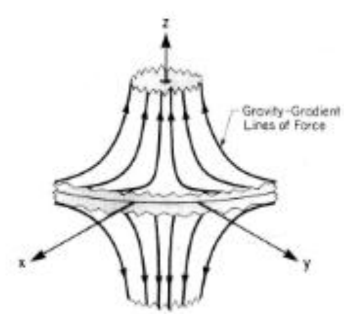
$$\vec{g}(\vec{r}) = \vec{g}(\vec{r}_o) + \nabla\vec{g}\,|_o \cdot\Delta(\vec{r}-\vec{r}_o) + o(\Delta(\vec{r}-\vec{r}_o))$$

Its evaluation can be conveniently achieved with dyadics

$$\nabla\vec{g}(\vec{r}) = \nabla(-\frac{\mu}{r^2}\hat{r}) = \nabla(-\frac{\mu}{r^3}\vec{r}) = 3\frac{\mu}{r^4}\vec{r}\hat{r} - \frac{\mu}{r^3}\nabla\vec{r}$$

$$\nabla\vec{r} = \nabla(x\hat{i} + y\hat{j} + z\hat{k}) = (\frac{\partial()}{\partial x}\hat{i} + \frac{\partial()}{\partial y}\hat{j} + \frac{\partial()}{\partial z}\hat{k})(x\hat{i} + y\hat{j} + z\hat{k}) = \hat{i}\hat{i} + \hat{j}\hat{j} + \hat{k}\hat{k} = \underline{\underline{u}}$$

$$\nabla\vec{g}(\vec{r}) = \frac{\mu}{r^3}(3\hat{r}\hat{r} - \underline{\underline{u}}) = \underline{\underline{G}}$$



Dyadics in Astrodynamics

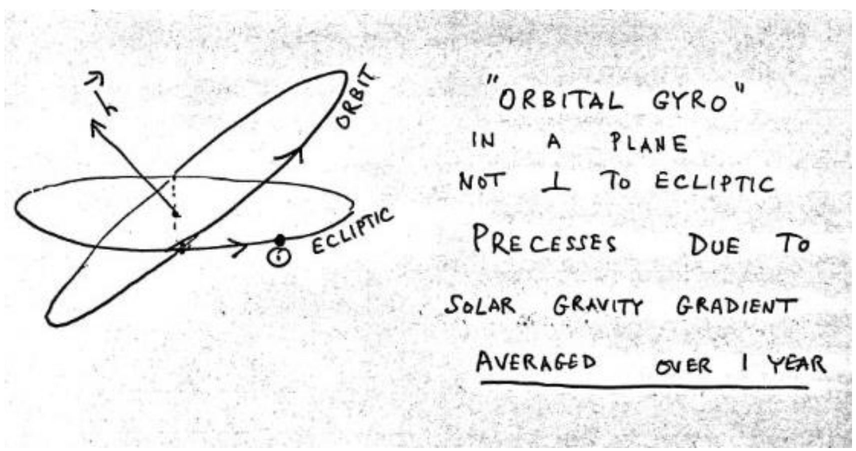
ORBITAL DYADIC	Wobble due to J2
GRAVITY GRADIENT DYADIC	Third body perturbation Wobble due to Sun and Moon Tidal effects Weak Stability Boundary Transfers Launcher Optimal Guidance
INERTIA DYADIC	Tethered Systems

Wobble Due To Third Body (Sun And Moon)

The third body perturbation effect is a secular one, due to Solar and Lunar gravity gradients, with the orbital axis preceding around the ecliptic axis or the Moon orbit plane axis.

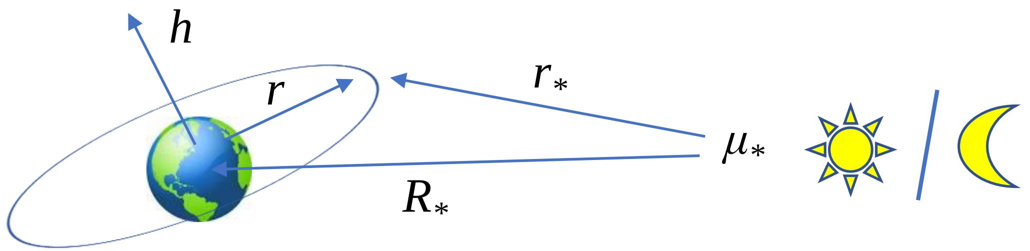
The effect can be averaged over 1 year (Sun) or 1 month (Moon).

This is a direct gravity gradient effect, easy to evaluate through dyadic formulation.



An original sketch by J.V. Breakwell

Third Body (Sun and Moon) Direct Effects



$$\frac{d\vec{h}}{dt} = \vec{r} \times \vec{f}_* = \vec{r} \times G_* \ \vec{r} = \vec{r} \times \frac{\mu_*}{R_*^3}\left(3\hat{r}_*\hat{r}_* - \underline{\underline{u}}\right)\cdot \vec{r} = 3\frac{\mu_*}{R_*^3}\hat{r}_* \cdot \vec{r} \ \vec{r} \times \hat{r}_*$$

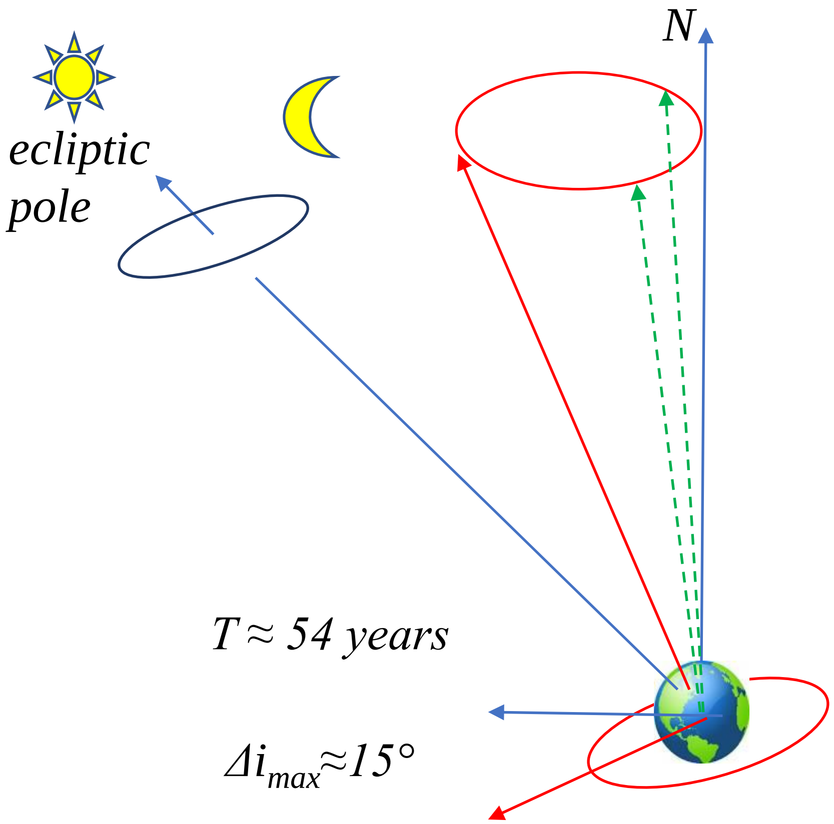
Average along S/C orbit $\left\langle \frac{d\vec{h}}{dt} \right\rangle = \frac{3}{2}\frac{\mu_* r^2}{R_*^3}\hat{R}_* \cdot \left(\underline{\underline{u}} - \hat{h}\hat{h}\right) \times \hat{R}_* = -\frac{3}{2}\frac{\mu_* r^2}{R_*^3}\hat{R}_* \cdot \hat{h}\hat{h} \times \hat{R}_*$

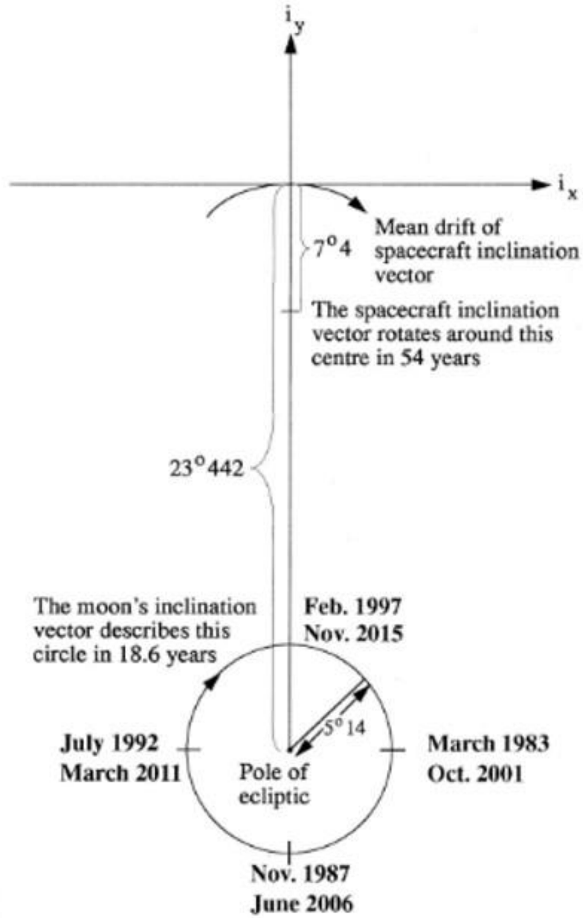
Average in third body period $\left\langle\left\langle \frac{d\vec{h}}{dt} \right\rangle\right\rangle = -\frac{3}{4}\frac{\mu_* r^2}{R_*^3}\hat{h} \cdot \hat{h}_* \ \hat{h}_* \times \hat{h}$

Analogy to Poisson $\frac{d\vec{h}}{dt} = \dot{\vec{h}} + \vec{\omega} \times \vec{h}$ $\quad \left\langle\left\langle \vec{\omega}_* \right\rangle\right\rangle = -\frac{3}{4}\frac{\mu_*}{R_*^3}\frac{r^2}{h}\hat{h}_* \cdot \hat{h} \ \hat{h}_*$

Station Keeping Inclination Maneuvers for GEO

Combined J2, Solar and Lunar perturbations affect GEO platforms, dictating a North-South correction maneuver at node to limit inclination.

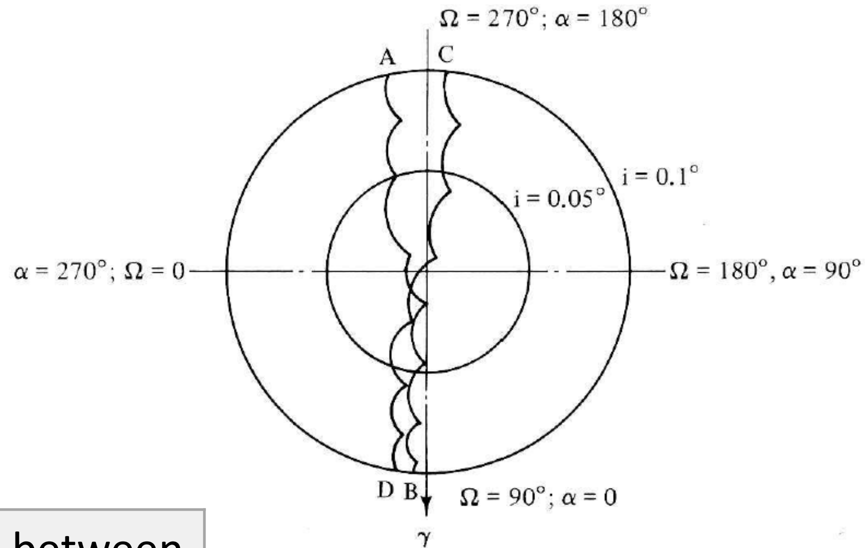




E.M. Soop "Handbook of Geostationary Orbits"

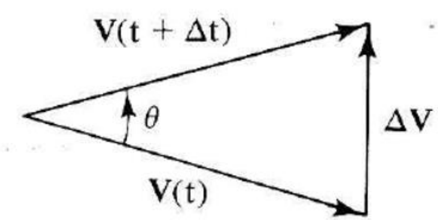
Station Keeping Inclination Maneuvers for GEO

A different view about periodic inclination corrections:



Inclination Limit (deg)	ΔV per maneuver (m/s)	Average Time between maneuvers (days)
0.01	1	10

$$\Delta v = 2v \sin\Delta i$$



B. N. Agrawal "Design of Geosynchronous Spacecraft"

Dyadics in Astrodynamics

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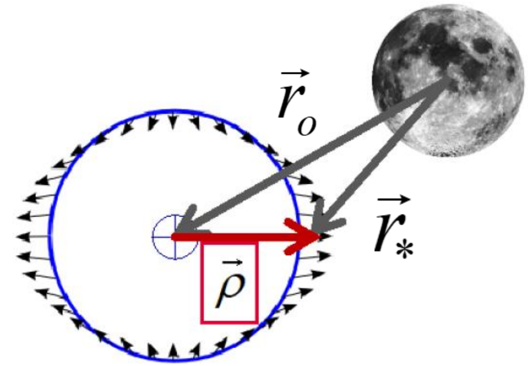
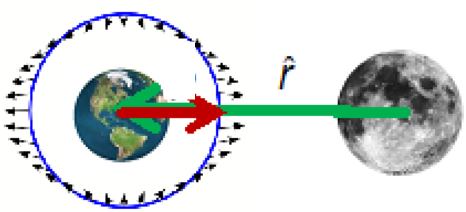
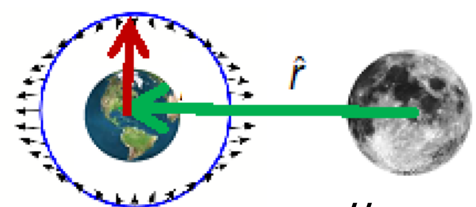
Gravity Dyadic as Tide Tensor

$$\vec{a}(\vec{r}_*) - \vec{a}(\vec{r}_o) = \nabla\vec{g}\,|_o \cdot \Delta(\vec{r}_* - \vec{r}_o) = \frac{\mu_*}{r_o^{\,3}}(3\hat{r}_o\hat{r}_o - \underline{\underline{u}}) \cdot \vec{\rho}$$

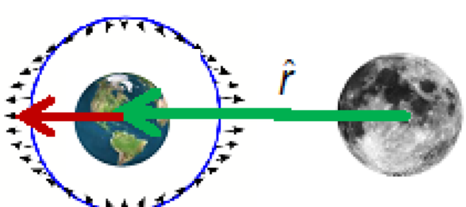




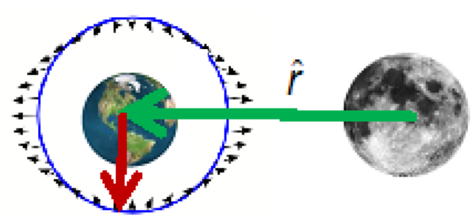
$$\vec{a} = \frac{2\mu_*}{r_o^{\,3}}\vec{\rho}$$



$$\vec{a} = -\frac{\mu_*}{r_o^{\,3}}\vec{\rho}$$



$$\vec{a} = \frac{2\mu_*}{r_o^{\,3}}\vec{\rho}$$



$$\vec{a} = -\frac{\mu_*}{r_o^{\,3}}\vec{\rho}$$

Sun and Moon Indirect (Tidal) Effects

SUN AND MOON

↓

DEFORMATION ON THE EARTH

↓

change in the distribution of the EARTH mass

↓

variations in the gravitational potential (TIDAL POTENTIAL)
proportional to the elastic properties of the Earth
Modeled by Love Numbers

↓

Tidal orbital perturbations

$$U_*(\vec{r}) = \frac{\mu_*}{r_*}\left(\frac{r}{r_*}\right)^2 S_2$$

$$U_*(R_\oplus) = \frac{\mu_*}{r_*}\left(\frac{R_\oplus}{r_*}\right)^2 S_2$$

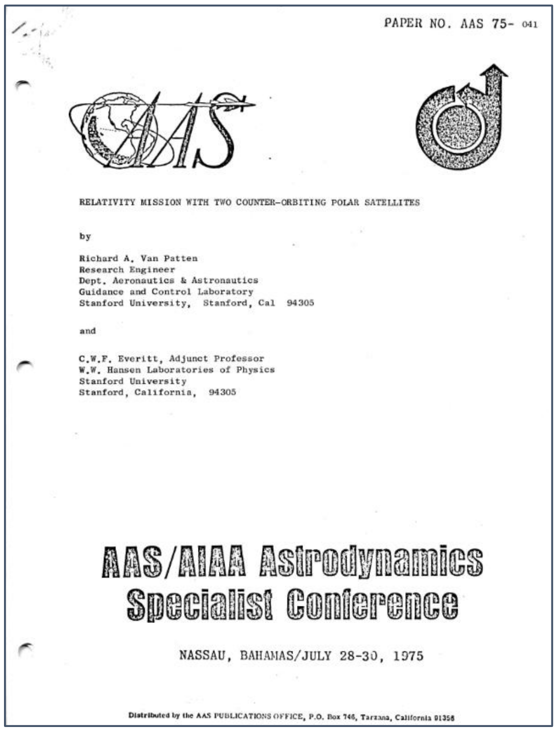
$$T(R_\oplus) = K_2 \frac{\mu_*}{r_*}\left(\frac{R_\oplus}{r_*}\right)^2 S_2$$

$$T(r) = K_2 \frac{\mu_*}{r_*}\left(\frac{R_\oplus}{r_*}\right)^2 \left(\frac{R_\oplus}{r}\right)^3 S_2 = K_2 \mu_* \frac{R_\oplus^5}{r_*^3 r^3} S_2$$

Sun and Moon Tidal Effects

Small variations are observable through high accuracy measurements, like the ones proposed by:

- Two counter-orbiting polar satellites (*Stanford, 1975, Breakwell-Van Patten-Everitt*)
- LARES Project (*Scuola di Ingegneria Aerospaziale, 2012, Paolozzi-Ciufolini*)



PAPER NO. AAS 75-041

AAS

RELATIVITY MISSION WITH TWO COUNTER-ORBITING POLAR SATELLITES

by

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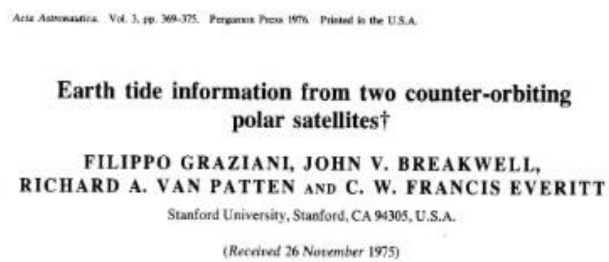
AAS/AIAA Astrodynamics Specialist Conference

NASSAU, BAHAMAS/JULY 28-30, 1975

Distributed by the AAS PUBLICATIONS OFFICE, P.O. Box 746, Tarzana, California 91356

The relativity experiment requiring two counter-orbiting spacecraft (1975).



Acta Astronautica. Vol. 3, pp. 369-375. Pergamon Press 1976. Printed in the U.S.A.

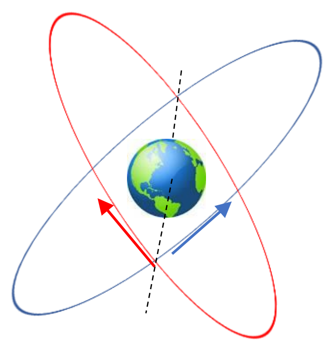
Earth tide information from two counter-orbiting polar satellites†

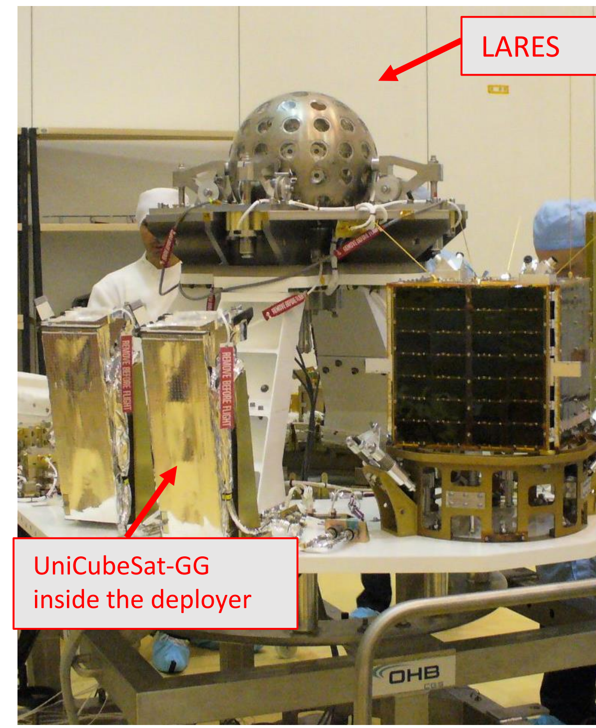
FILIPPO GRAZIANI, JOHN V. BREAKWELL, RICHARD A. VAN PATTEN AND C. W. FRANCIS EVERITT

Stanford University, Stanford, CA 94305, U.S.A.

(Received 26 November 1975)

The 1975 paper proposing the dual-use to investigate tidal effects, co-authored by JVB.





The real mission to study tidal effects with the involvement of the School of Rome (2012).

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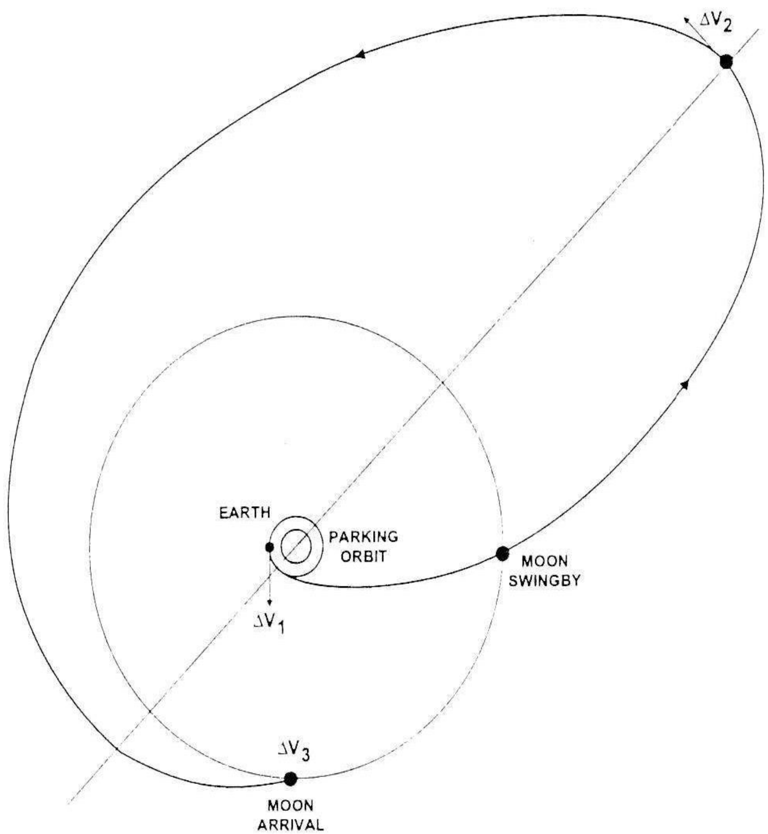
Weak Stability Boundary Orbits

- Mission analysis has been developed for decades on the basis of **two body problem** models with perturbations (planetary missions) and patched two body problem (interplanetary missions).
- More recently, the WSB method has been used to analyse interplanetary missions in the context of **restricted three body problems with the fundamental effect of a fourth attracting body** to accomplish the mission (such as Earth-Moon-Sat plus Sun). The result is a bigger flexibility and extension of launch windows and propellant saving at the cost of longer duration missions.

References:

- **J. Kawaguchi** et al., “ON MAKING USE OF LUNAR AND SOLAR GRAVITY ASSISTS IN LUNAR-A, PLANET-B MISSIONS” *Acta Astronautica Vol. 35. No. 9-l I, pp. 633-642, 1995*.
- **E.A. Belbruno** et al., “CALCULATION OF WEAK STABILITY BOUNDARY BALLISTIC LUNAR TRANSFER TRAJECTORIES” *AIAA 2000-4142, AIAA/AAS Astrodynamics Specialist Conference, 2000*.
- **M. Bello Mora** et al., “A SYSTEMATIC ANALYSIS ON WEAK STABILITY BOUNDARY TRANSFERS TO THE MOON” *IAF-00-A.6.03, 51st International Astronautical Congress, 2000*.
- **P. Teofilatto** et al., “ON THE DYNAMICS OF WEAK STABILITY BOUNDARY LUNAR TRANSFERS” *Celestial Mechanics and Dynamical Astronomy 79: 41–72, 2001*.

An Example:



- The spacecraft is injected into a circular parking orbit.

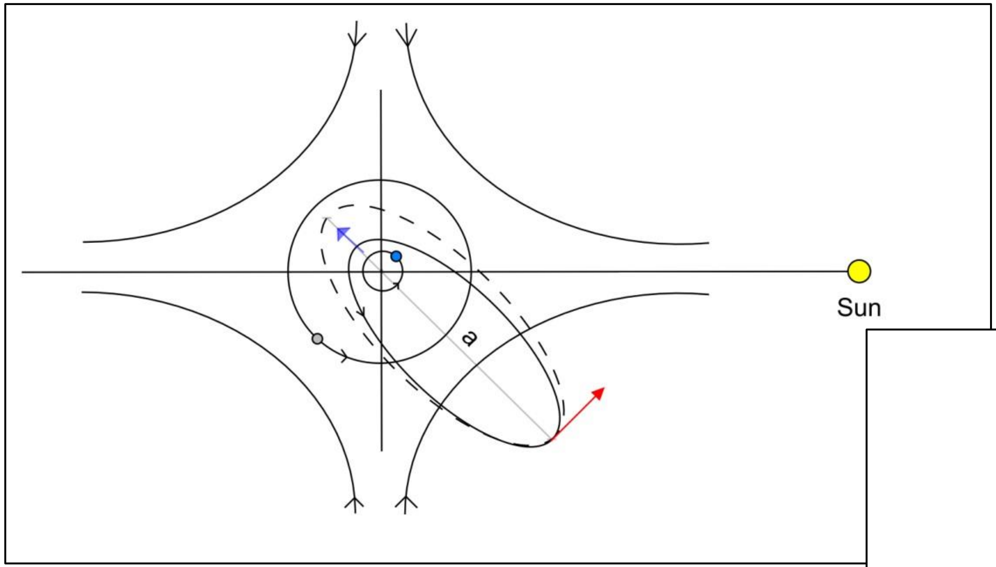
- A perigee kick manoeuvre **ΔV_1** injects the spacecraft into a lunar transfer orbit.

- An unpowered **Moon swingby** injects the spacecraft into a high eccentric orbit with apogee of about 1.4 million km in WSB-Earth.

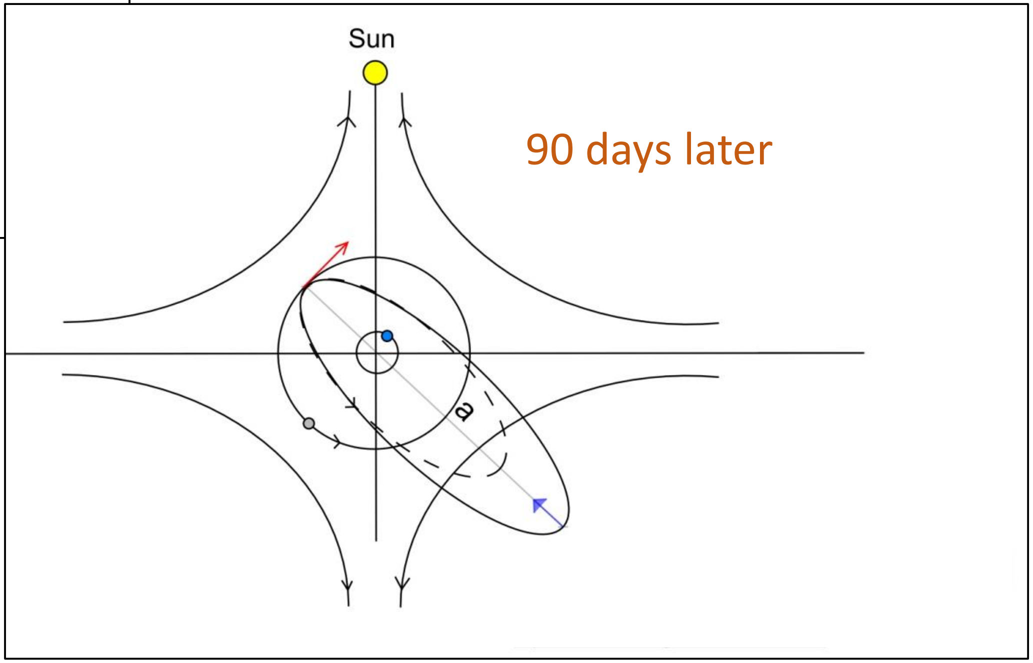
- A second manoeuvre **ΔV_2** introduces a small correction in order to inject the spacecraft into a trajectory which joins the apogee and the Moon orbit.

- When the spacecraft approaches the Moon, both Earth (gravity gradient effect of the Earth) and Sun (gravity gradient effect of the Sun) contribute to the **lunar capture**.

Sun Gravity Gradient Effects



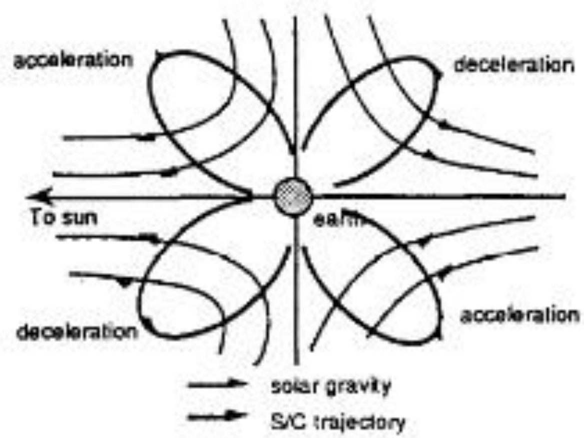
Gravity gradient acts at the orbit aphelion
Semi-major axis (a) increases
perihelion height increases

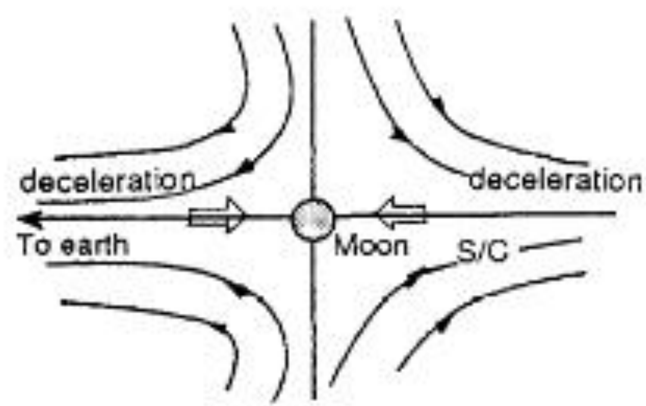


90 days later

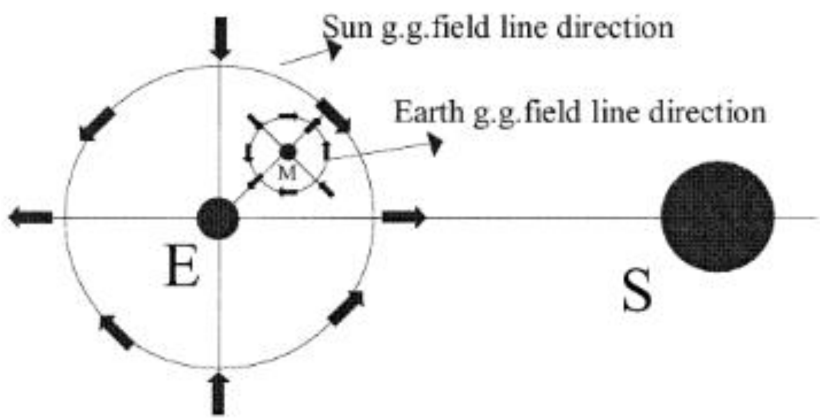
Gravity gradient acts at the orbit perihelion
Semi-major axis (a) decreases
Aphelion height decreases

Gravity gradient exploited in WSB





J. Kawaguchi et al., "ON MAKING USE OF LUNAR AND SOLAR GRAVITY ASSISTS IN LUNAR-A, PLANET-B MISSIONS"



Indeed, gravity gradient offers a useful explanation of the advantage of the long transfers.

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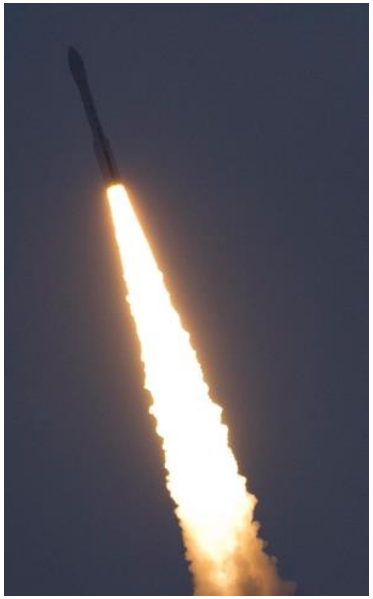
Optimal Guidance of Launchers



Functional
$$J = \varphi + \int_0^T L(x,u,t)\,dt$$

Constrained by dynamics
$$\dot{x} = f(x,u,t)$$

Introducing Lagrangian multipliers
$$J' = \int_0^T L(x,u,t)\,dt + \int_0^T \lambda(t)\left[f(x,u,t) - \dot{x}\right]dt$$

Defining the Hamiltonian
$$H = L + \lambda f$$

and integrating by part
$$J' = \varphi + \int_0^T \left(H(x,u,\lambda) + x\dot{\lambda}\right)dt$$

Leading to Pontryagin conditions
$$\dot{\vec{x}} = \frac{\partial H}{\partial \vec{\lambda}} \qquad \dot{\vec{\lambda}} = -\frac{\partial H}{\partial \vec{x}} \qquad \frac{\partial H}{\partial \vec{u}} = 0$$

Optimal Guidance of Launchers (2)

$$\begin{cases} \dot{\vec{r}} = \vec{v} \\ \dot{\vec{v}} = \vec{g}(\vec{r}) + \dfrac{\vec{T}}{m} \\ \dot{m} = -\dfrac{T}{g_o I_{sp}} \end{cases}$$

In our case:
Cost function → $J = m_{final}$
Dynamics (← the system above)
Hamiltonian ↓

$$H = -\ \lambda_m \frac{T}{g_o I_{sp}} + \vec{\lambda}_r \cdot \vec{v} + \vec{\lambda}_v \cdot \left(\vec{g}(\vec{r}) + \frac{T}{m} \frac{\vec{u}}{u} \right)$$

Pontryagin conditions become:

$$\frac{\partial H}{\partial \vec{r}} = \vec{\lambda}_v \cdot \nabla \vec{g} \qquad \frac{\partial H}{\partial \vec{v}} = \vec{\lambda}_r \qquad \frac{\partial H}{\partial m} = \vec{\lambda}_v \cdot \frac{\vec{T}}{m^2} \qquad \frac{\partial H}{\partial \vec{u}} = \vec{\lambda}_v \cdot \left(\frac{\partial}{\partial u} \left(\frac{T\hat{u}}{m} \right) \right) = 0$$

If T/m is constant, the Lawden primer vector behavior is conveniently described by dyadics

$$\vec{\lambda}_v \cdot \left(\underline{\underline{U}} - \hat{u}\hat{u} \right) = 0 \longrightarrow \hat{\lambda}_v = \hat{u}$$

An Application:

Lawden primer vector

$$\ddot{\vec{\lambda}}_v = \underline{\underline{G}} \cdot \vec{\lambda}_v$$

within the simplifying assumption of flat Earth and constant gravity (applicable to students sounding rockets), it follows:

$$\ddot{\vec{\lambda}} = 0$$

The classical Bilinear tangent law

$$\vec{\lambda} = \vec{a} + \vec{b}t \qquad \hat{u}_{opt} = \frac{\vec{a} + \vec{b}t}{\left|\vec{a} + \vec{b}t\right|}$$

Dyadics in Astrodynamics

ORBITAL DYADIC	Wobble due to J2
GRAVITY GRADIENT DYADIC	Third body perturbation Wobble due to Sun and Moon Tidal effects Weak Stability Boundary Transfers Launcher Optimal Guidance
INERTIA DYADIC	Tethered Systems

Inertia Dyadic

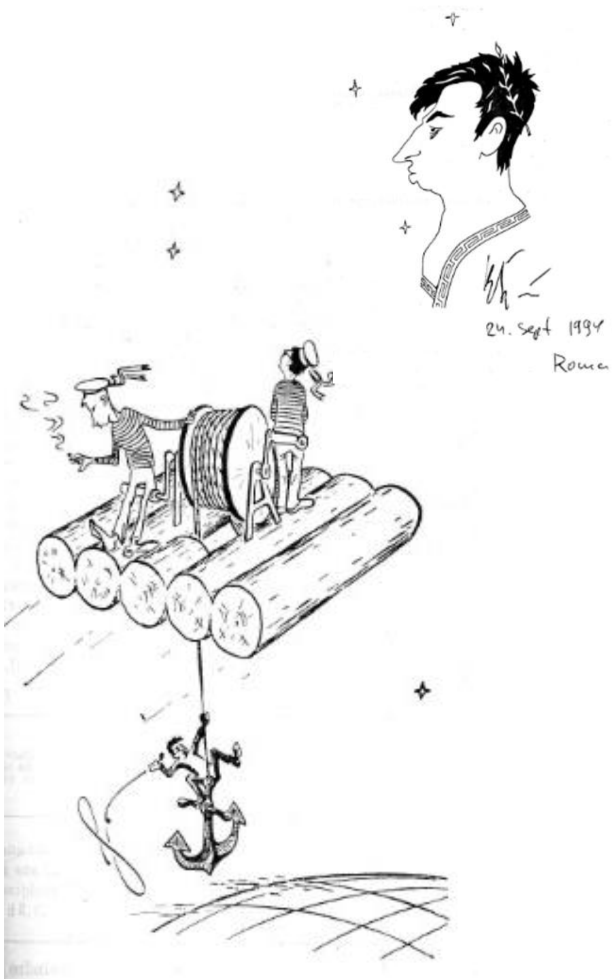
The dyadic notation is particularly well suited to the attitude motion of a rigid body (attitude dynamics) since the inertia is described most easily by a dyadic:

$$I = A\hat{i}\hat{i} + B\hat{j}\hat{j} + C\hat{k}\hat{k}$$

The gravity torque acting on a body is:

$$\vec{M} = \frac{3\mu}{r_0^3}\hat{r}_0 \times \int_M \left(\rho^2 \mathbf{u} - \vec{\rho}\vec{\rho}\right)\, dm \cdot \hat{r}_0$$

$$\vec{M} = \frac{3\mu}{r_0^3}\hat{r}_0 \times \mathbf{I}\cdot\hat{r}_0$$



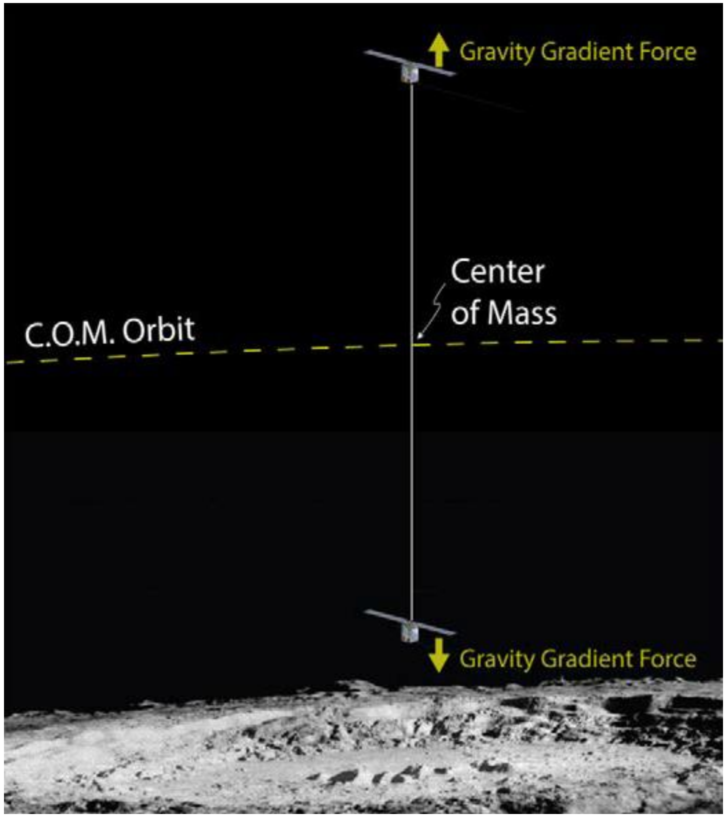
A sketch from Beletskii
"Essais sur le mouvement des corps cosmiques"

Dyadics (Attitude)

The classical application is offered by gravity gradient stabilization (one more chance to recall Stanford's AeroAstro contribution!).

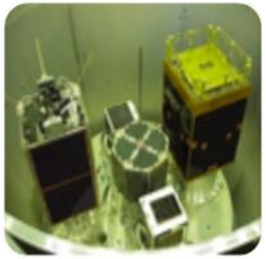
Gravity gradient – among many other cases of interest – may be also applied to a proposed NASA-Goddard mission called BOLAS.

Two 12U CubeSats linked by a 112 miles long tether flying in a very low altitude lunar orbit.



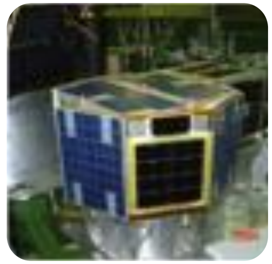
Thinking about such a mission allows to move to small platforms…

Microsatellites launched by GAUSS



UNISAT
2000

UNISAT-2
2002

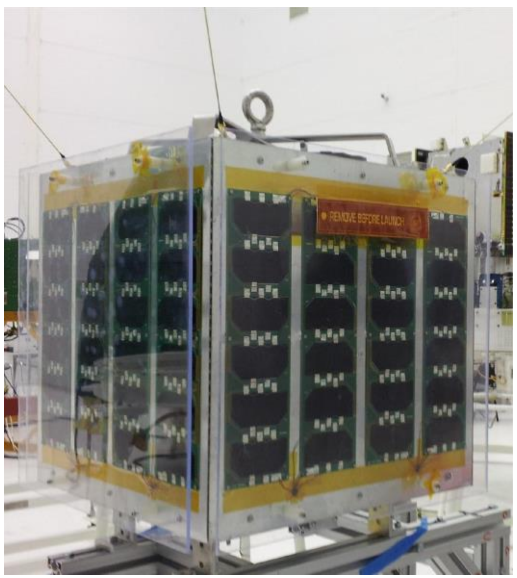


UNISAT-3
2004

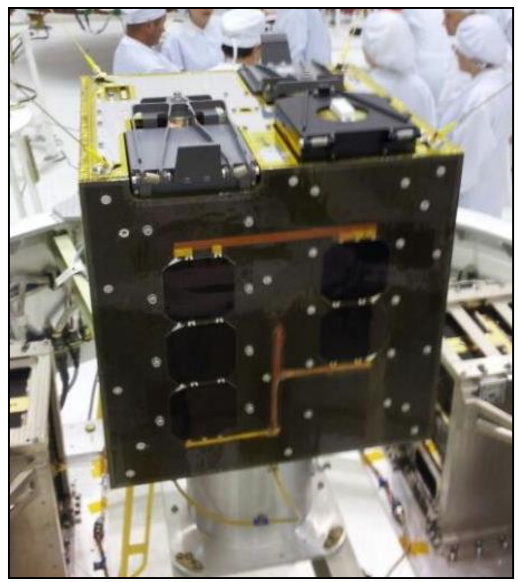
UNISAT-4
2006

EDUSAT
2011

UNICUBE-
GG 2012



UNISAT-5
2013



UNISAT-6
2014

TUPOD
2017

The Experience of GAUSS

- Along the years of studying and teaching, the possibility of real applications has been always stressed, together with the support to young scholars and the chance and the challenge of team work.

- Researches carried out at University of Rome through GAUSS (*Gruppo di Astrodinamica Università degli Studi la Sapienza*).

- 2012: the Group became the private company GAUSS (***Group of Astrodynamics for the Use of Space Systems***).

- Along the years GAUSS grew up….

More to come….

International Moon Mission

Fregat
Upper Stage
provides with
acceleration
to the Moon
escape trajectory

Soyuz-2
Launch Vehicle
provides launching
into the parking
orbit

- Spacecraft **mass**: about 25 kg.
- Spacecraft may be launched into escape trajectory to Moon as a **piggy-back** payload.
- As launch vehicle, a potential candidate may be the Russian **Soyuz/Fregat**. The Fregat upper stage will provide acceleration from parking orbit to the escape trajectory to Moon (ΔV = 3150 m/s), then the payload will be separated and it will continue its autonomous flight.

A microsatellite to the Moon (and beyond): A dream…

This is the mission we are trying to achieve.
It is a dream… but sometimes dreams come true.

Credit: SpaceX